

# A Novel Method Describing the Space Charge Limited Region in a Planar Diode

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## نگاهی جدید در توصیف فضای بار در دیود صفحه‌ای

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### چکیده

در این مقاله روش نوینی برای توصیف فیزیکی ناحیه فضایی بار در یک دیود صفحه‌ای ارائه شده است. با توجه به این که در این روش از رویکرد حوزه زمان استفاده شده است، رفتار گذرای مسئله قبل از رسیدن به حالت پایدار به عنوان نتیجه‌ای از روش قابل دسترسی است. با فرض ورود الکترون‌ها با سرعت اولیه صفر، توزیع پتانسیل و جریان بدست آمده از این روش تطابق کامل با نتایج Child دارند. همچنین، اعمال روش برای ورود الکترون با سرعت غیرصفر نیز نتایجی موافق با کارهای قبلی دارد.

**Abstract** — A novel and rather simple method is presented to describe the physics of space-charge region in a planar diode. The method deals with the issue in the time domain and as a consequence transient time behavior can be achieved. Potential distributions and currents obtained using this technique, supposing zero initial velocity for

electrons, reveal absolute agreement with Child's results. Moreover, applying the method for non-zero uniform initial velocity for electrons, gives results which are in good agreement with previous works.

*Index Terms* — space-charge region, space-charge limited current, planar diode, initial velocity.

## I. INTRODUCTION

Making different assumptions, the characteristics of space-charge limited region in a planar diode have been studied. The most basic supposition having been made is to consider the electrodes infinitely large and therefore to consider the equipotential surfaces as planes parallel to the electrodes. Based on this presumption and also assuming zero velocity for the electrons emitted, Child [1] and Langmuir [2] obtained the well known three halves law and a formula for potential distribution.

Langmuir and Blodgett followed the same procedure, however with different geometries. In [3], they calculated space charge between coaxial cylinders, and in [4] they calculated it for Concentric Spheres.

Extending the classical one dimensional Child-Langmuir law to two dimensions has been done in [5-7]. Two-dimensional space-charge limited current in cylinders with finite length emitter has been studied by Kostov and Barroso [8]. In [9] the analytic and computational extensions to the one-dimensional Child–Langmuir law was reviewed. Koh etc, presented a 3D model of Child-Langmuir law for hot electron emission in planar and cylindrical gap, including finite emission energy effects [10].

In [11-13], considering the interactions between electrons, quantum extensions of Child-Langmuir law were studied.

An experimental investigation of space-charge-limited flow of current in a nanogap was presented in [14]. When distance is comparable to wavelength, the current density varies as square root of applied voltage, unlike the classical Child-Langmuir law [14]. Exact relativistic solutions for the one-dimensional space-charge-limited diode was given in [15].

Effects of short pulses and THz frequencies are studied and presented in [16-18]. In [19], classical Child-Langmuir law was extended to the Coulomb blockade regime.

Studying the space charge current, assuming Maxwellian distribution for initial velocity of electrons was studied by Fry and Langmuir [20,21]. Through a simplified theory, Parsons presented an Exact integration of the space charge equation in a planar diode [22]. The effects of finite initial velocity was examined in [23,24].

As mentioned above, extensive and continued suervy has been carried out on space charge. These efforts which last over a century shows the importance of the topic.

Space charge, is also an important factor in other fields of physics, like beam dynamics of cyclotrons and other particle accelerators (e.g. [25,26])

Realizing this, we have proposed a different approach describing the space-charge limited region. The method is consistant with physics of the problem and simple to apply. The method can be applied for non-zero initial velocity. Moreover, transient results can be achieved.

Modeling the problem with entering planes is nearly a usual way, e.g. [19]. In [19] another aspect of the problem, which is finding a threshold voltage for single electron injection is addressed.

## II. FORMULATION

The situation is schematically illustrated in Fig. 1.

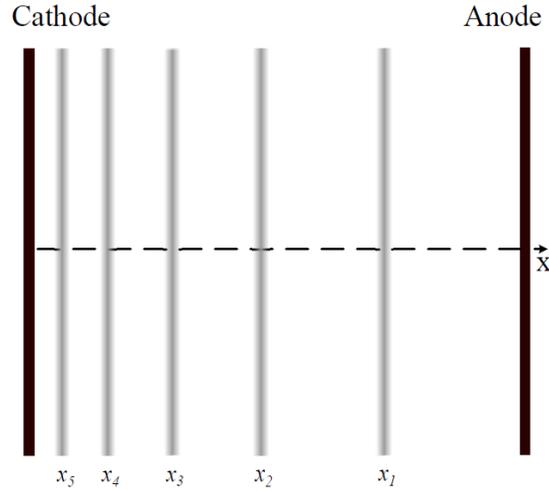


Fig. 1. The electron beam modeled by plates of electric charge entering the space.

The electron beam is modeled by plates of electric charge periodically entering the space between the electrodes. The positions of the charge plates are represented by  $x_1, x_2, \dots$ . The plates are allowed to move only in the  $x$  direction and all have the same initial velocity  $u_0$ . The cathode is at potential zero and the anode is at potential  $V_0$ . As the charge plates enter the inter-electrode space, the presence of each plate alters the electric field in the space. Assuming infinitely large plates and charge density  $\sigma_{ins}$  on them, the difference between electric field on the right and the left of the plates is  $\sigma_{ins}/\epsilon_0$ , where  $\epsilon_0$  is the vacuum permittivity. The emission of the electrons is modeled by periodic entrance of plates so that we would be able to observe the transient behavior. The transient behavior of the system is described as follows:

Before emission starts, there are no electrons in the space, the potential distribution is linear and the electric field is constant between the electrodes. The entrance of the first group alters the electric field. Motion of the plate is determined according to the force imposed by the electric field. As the next plate enters, the electric field gets modified. The two plates now move in accordance with the modified electric field. The process goes on, plates enter periodically, electric field changes and their position alter correspondingly. The plates enter the space until the system reaches the steady state. In this state the positions of the available plates in the space are almost fixed and the space charge has been formed.

Consider  $N$  to be the number of charge plates in the space. As the first plate enters  $N$  is 1. After the presence of the second plate  $N$  would be 2. As the plates enter  $N$  increases until the system reaches steady state. From this point

on, the entrance of plates into the space would not raise  $N$ . Imagine at a time there are  $N$  plates in the space. The electric field between  $x_{n-1}$  and  $x_n$ , denoted by  $E_n$ , is calculated as:

$$E_n = (-\sigma - \frac{x_1 + \dots + x_N}{d} \sigma_{ins} + (n-1)\sigma_{ins})/\epsilon_0$$

$$n = 1, 2, \dots, N \quad (1)$$

where  $\sigma$  is the charge density on the electrodes before the entrance of any plate and  $d$  is the distance between the electrodes. Using Newton's second law (equation (2)) and equation (3), the equation of motion of the  $n^{\text{th}}$  plate can therefore be described as equation (4)

$$F_n = m\ddot{x}_n \quad (2)$$

$$F_n = -eE_n \quad (3)$$

$$\ddot{x}_n = \frac{e}{m\epsilon_0} (-\sigma - \frac{x_1 + \dots + x_N}{d} \sigma_{ins} + (n-0.5)\sigma_{ins})$$

$$n = 1, 2, \dots, N \quad (4)$$

where  $F_n$  is the applied force and  $E_n$  is the applied electric field on the electron.  $e$  and  $m$  are the charge and mass of electron, respectively.

As the plates enter, their position is calculated using equation (1). The current can be calculated from their arrival on the anode. The total current density between the arrivals of the  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  plate is composed of two components:  $J_1$  which is due to the arrival of a plate with charge density  $\sigma_{ins}$  on the anode and  $J_2$  the displacement current. These components are calculated as follows:

$$J_1 = \frac{\sigma_{ins}}{t_{arr,n+1} - t_{arr,n}}. \quad (5)$$

where  $t_{arr,n}$  and  $t_{arr,n+1}$  are the arrival times of the  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  plate, respectively.

$$J_2 = \frac{(\frac{\sum x_i}{d} \sigma_{ins})_{t_{arr,n+1}} - (\frac{\sum x_i}{d} \sigma_{ins})_{t_{arr,n}}}{t_{arr,n+1} - t_{arr,n}}. \quad (6)$$

where, in the nominator, the summations hold for available  $x_i$ s in the space at times  $t_{arr,n+1}$  and  $t_{arr,n}$ .

At steady state, the  $x_n$ s would remain unchanged,  $J_2$  approaches zero and the current density would actually be  $J_1$ . The potential at the steady state can be obtained integrating the electric field.

In addition to the above mentioned way, the current can be obtained from the below formula:

$$J = \rho v. \quad (7)$$

We expect that in steady state,  $\rho v$  would be the same for all points and be the same as  $J_1$ . The results presented in section VI live up to this expectation.

### III. PROCEDURE

The procedure of the method is as follows:

First plane is emitted from the cathode and enters the space. The motion of this plane is described by equation(4) where  $n$  is equal to  $1$ . Using Laplace transform the position and velocity of this plane at  $T$  seconds later when, the second plane enters, is obtained. As the second plane enters, the electric fields and consequently the forces applied to planes change. Making use of equation(4) and Laplace transform, the positions and velocities of these two planes at the time of the arrival of the third plane are determined. The procedure goes on this way. Planes enters into the space one after another and their positions and velocities are being calculated. As the planes collide the anode, the arrival current can be calculated using equation(5). Displacement current is calculated using equation(6). Total current is the sum of these two current.

### IV. COMPARISONAL RESULTS

As an illustrative example, the technique presented in the previous section has been applied to a system with  $V_0 = 0.1 V$ ,  $d = 5\mu m$ ,  $T = 21ns$ ,  $\sigma_{ins} = 0.12\sigma$  and zero initial velocity for emitted electrons. The normalized potential distribution and current density, are shown in Fig. 2 and Fig. 3, respectively.

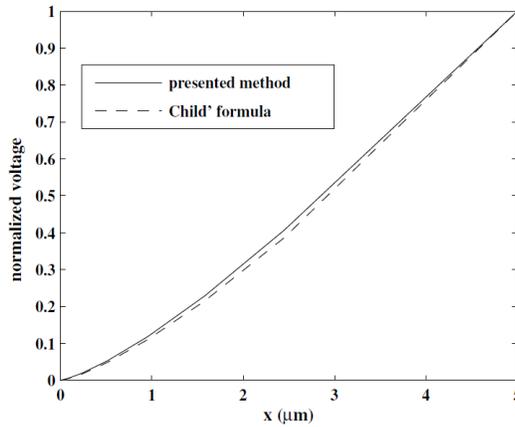


Fig. 2. Normalized potential distribution obtained from Child's method vs presented method.  $V_0 = 0.1 V$ ,  $d = 5\mu m$ ,  $u_0 = 0$ .

To be able to make a comparison, the potential distribution obtained from Child's formula is also drawn in Fig. 2. Clearly, the two plots are almost congruent.

Child-Langmuir formulas for space-charge limited current and potential are given by[1][2]:

$$I_{CL} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} V_0^{3/2} \quad (8)$$

$$V_{CL}(x) = \left(\frac{x}{d}\right)^{4/3} V_0, \quad (9)$$

For the above system  $I_{CL}$  is 73 nA. As can be seen in Fig. 3, the current at the steady state reaches  $I_{CL}$ .

To consider the effect of non-zero initial velocity, the technique is applied to a system with the above parameters, but non-zero initial velocity equivalent to 0.1mV for emitted electrons. Using either of the two mentioned ways, the current at steady state would be 81.3nA.

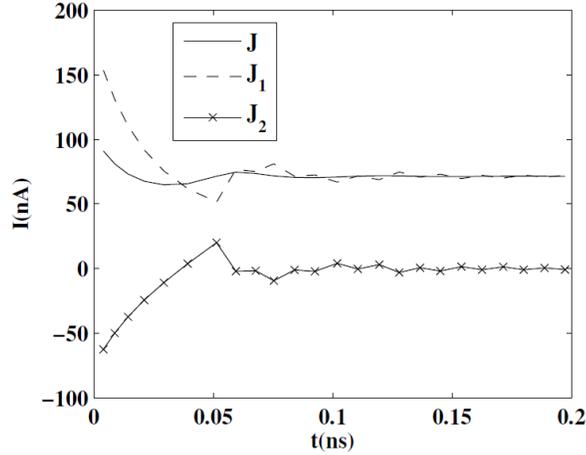


Fig. 3. Current density  $J$  and components  $J_1$  and  $J_2$   $V_0 = 0.1$  V,  $d = 5\mu\text{m}$ ,  $u_0 = 0$ .

In [23] the space-charge limited current with non-zero initial velocity is shown to be:

$$I_{SCL} = I_{CL} \left[ \left( \frac{mu_0^2}{2eV_0} \right)^{1/2} + \left( 1 + \frac{mu_0^2}{2eV_0} \right)^{1/2} \right]^3 \quad (10)$$

Making use of the above equation, space-charge limited current would be 81.1 nA, showing good agreement with the result of our method.

## V. CONCLUSION

In this paper, the electron beam was modeled by plates of electric charges. Applying the method gave reasonable results in agreement with previous works. The advantage of the method lies in the fact that it is simple and provides transient time results. The method can also be applied for non-zero initial velocity emission.

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