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Research note

Investigation of three-body $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays

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Abstract

We analyze three-body decays of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$. Under the factorization approach, there are tree level diagrams for these decay modes and the transition matrix element of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ decay is factorized into a $B^+ \to K^+ \bar{K}^0$ form factor multiplied by $\bar{D}^*(2007)^0$ decay constant and $B^+ \to \bar{D}^*(2007)^0$ form factor multiplied into $0 \to K^+ \bar{K}^0$ weak vertices form factor. The transition matrix element of $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decay is also factorized into a $B^0 \to D^*(2010)^-$ form factor multiplied into $0 \to K^+ \bar{K}^0$ weak vertices form factor. We investigate these decays by using the Dalitz plot technique. First, we use the general form of this technique and calculate the branching ratios of the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays and obtain the values 4.862×10^{-3} of and 49.212×10^{-4} , while the experimental results are less than 1.060×10^{-3} and 4.700×10^{-4} respectively. Then we assume that, in the $B \to D^* KK$ decays, because the mass of the D^* is too heavy against the K-mesons.So the momentum of the D^* can have a small amount. Namely, the D^* carries small momentum. In the probability summation of the amplitudes, the momentum of the D^* , instead of starting from its maximum value, starts from a state in which two light mesons make an angle of105 degrees. Using our assumption, we calculate the branching ratios of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays and obtain 0.475×10^{-3} and 4.677×10^{-4} respectively. In addition, we estimate the branching ratios for different angles between the two light mesons and get that when the angle between them is 180° , in fact they decay back to back, and these values become 0.318×10^{-3} and 1.726×10^{-4} , which are the best answers close to the experimental values. The branching ratios obtained by applying our assumption are compatible with the experimental results.

Keywords: B meson decays, factorization, dalitz plot analysis

1. Introduction

The three-body decays of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ were originally measured by BELLE Collaboration [1] and later on tabulated by the Particle Data Group (PDG) [2]. Long time ago, the $D \to K\pi\pi$ decay has been studied using the Dalitz plot technique [3]. According to this technique, we can find many articles such as [4-8] which don't assume that the mass of the K-meson is heavy in front of the pi-mesons. So the momentum of the K-meson is sizable against the momentum of the pi-mesons. In these kinds of three-body decays, whereas three particles are light, given that the theoretical momentums of the output particles are not directly calculable, momentums and form factors are written in terms of $s = (p_B - p_3)^2$ and $t = (p_B - p_1)^2$ [9,10], and Dalitz plot role should be used for calculation of the decay rate integral from $s_{\min}, t_{\min}$ to $s_{\max}, t_{\max}$. It seems that, when one of the final state particles is much heavier than the other two, in the general form of this technique, because the integral of the decay rate is taken over all angles between the momentums of the output particles, the results obtained by using the Dalitz plot model are much larger than the experimental value. As in our selected decays, we obtain $BR(B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0) = 4.862 \times 10^{-3}$ and $BR(B^0 \to D^*(2010)^- K^+ \bar{K}^0) = 49.212 \times 10^{-4}$ by using the general form of the Dalitz plot technique, which are much larger than the experimental values. In fact, in $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$

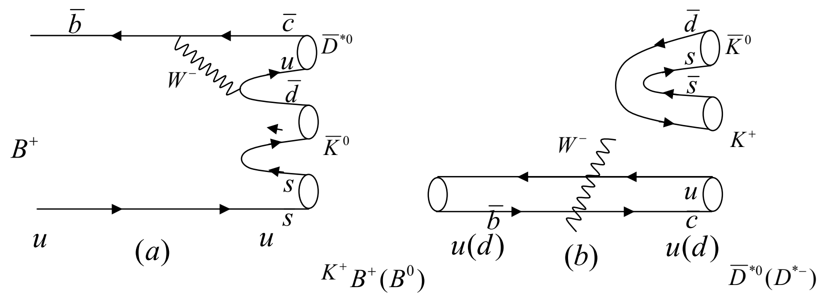

Figure 1. Quark diagram illustration the process $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays.

decays, such as $B \to J/\psi\pi\pi$ [11] and $B \to DKK$ decays [12], because the mass of the D^* is too heavy against the K mesons, the D^* carries a small momentum. This represents the fact that in the probability summation of the amplitudes, the momentum of the D^*, instead of starting from its maximum value (in the rest frame of the light mesons), starts from a state in which two light mesons make an angle of 105 degrees. With this assumption, we get $BR(B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0) = 0.475 \times 10^{-3}$ and $BR(B^0 \to D^*(2010)^- K^+ \bar{K}^0) = 4.677 \times 10^{-4}$ which are in good agreement with the experimental results. In addition, we estimate the branching ratios for different angles between the two light mesons and get that when the angle between them is 180°, in fact they decay back to back and the D^* is at rest, the best responses are obtained as $BR(B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0) = 0.318 \times 10^{-3}$ and $BR(B^0 \to D^*(2010)^- K^+ \bar{K}^0) = 1.726 \times 10^{-4}$

The direct three-body decays of mesons in general receive two distinct contributions: one from the point-like weak transition and the other from the pole diagrams that involve three-point or four-point strong vertices [13]. Before giving the matrix elements for the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays, we discuss the parametrization of the decay constants and form factors which appear in the factorized form of the hadronic matrix elements. (i) In the case of $< B^+ \to K^+ \bar{K}^0 > \times < 0 \to \bar{D}^*(2007)^0 >$, panel (a) in figure 1, both K mesons are placed in the form factor. Indeed, two-meson matrix element transition of B meson is done to the $K^+ \bar{K}^0$ mesons. The $\bar{D}^*(2007)^0$ meson is placed in the decay constant, which the vector meson's decay constant, such as D^*, is expressed in terms of the matrix element $< D^* | \bar{d}\gamma_\mu c | 0 >= f_{D^*} m_{D^*} \varepsilon_{D^*}$ [14]. (ii) In the cases of $< B^+ \to \bar{D}^* > \times < 0 \to K^+ \bar{K}^0 >$ and $< B^0 \to D^{*-} > \times < 0 \to K^+ \bar{K}^0 >$ panel (b) in figure 1, we have two kinds of form factors, one is transition of B meson to vector meson of the $\bar{D}^*$, $F^{B\bar{D}^*}$, and another is the weak form factor, $F^{K^+\bar{K}^0}$ arising from the vacuum vertices.

Regarding the resonance contributions, it should be noted that the decay of $R^+ \to K^+ \bar{K}^0$ has not been observed in the experimental tests, therefore, we ignore them in our calculations.

2 Amplitude of the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decay

2.1 Dalitz plot analysis

In the factorization approach, Feynman diagrams for three body $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays are shown in figure 1. Under the factorization approach, the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ decay amplitude consist of $< B^+ \to K^+ \bar{K}^0 > \times < 0 \to \bar{D}^* >$ and $< B^+ \to \bar{D}^* > \times < 0 \to K^+ \bar{K}^0 >$ distinct factorizable terms, where $< B \to KK >$ denotes two-meson transition matrix element and $< 0 \to K^+ \bar{K}^0 >$ denotes weak interaction form factor. The matrix elements of the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ decay amplitude are given by

$$\left\langle \bar{D}^{*0} K^+ \bar{K}^0 \middle| H_{eff} \middle| B^+ \right\rangle$$
$$\propto a_2 \left\langle \bar{D}^{*0} \middle| (u\bar{c})_{V\text{-}A} \middle| 0 \right\rangle \left\langle K^+ \bar{K}^0 \middle| (d\bar{b})_{V\text{-}A} \middle| B^+ \right\rangle$$
$$+ a_1 \left\langle K^+ \bar{K}^0 \middle| (u\bar{d})_{V\text{-}A} \middle| 0 \right\rangle \left\langle \bar{D}^{*0} \middle| (c\bar{b})_{V\text{-}A} \middle| B^+ \right\rangle \quad (1)$$

and the $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decay amplitude consist of $< B^0 \to D^{*-} > \times < 0 \to K^+ \bar{K}^0 >$, so the matrix elements of the $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decay amplitude are given by

$$\left\langle D^{*\text{-}} K^+ \bar{K}^0 \middle| H_{eff} \middle| B^0 \right\rangle \propto$$
$$a_1 \left\langle K^+ \bar{K}^0 \middle| (u\bar{d})_{V\text{-}A} \middle| 0 \right\rangle \left\langle D^{*\text{-}} \middle| (c\bar{b})_{V\text{-}A} \middle| B^0 \right\rangle \quad (2)$$

For the current-induced process, the two-meson transition matrix element $< K^+ \bar{K}^0 | (d\bar{b})_{V-A} | B^+ >$, has the general expression as follows [9]

$$\left\langle K^+(p_1) \bar{K}^0(p_2) \middle| (d\bar{b})_{V\text{-}A} \middle| B^+ \right\rangle$$

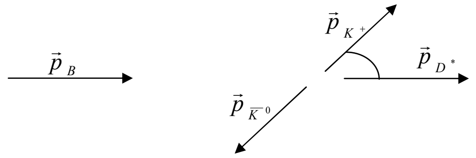

Figure 2. Definition of helicity angle θ, for the decay.

$$= ir(p_B - p_1 - p_2)_\mu$$
$$+ i\omega_+(p_2 + p_1)_\mu + i\omega_-(p_2 - p_1)_\mu \qquad (3)$$

where the two-meson transition form factors are computed from point-like and pole diagrams, and we need the strong coupling of $B^* B^{(*)} K$ and $BBKK$ vertices. The form factors r, ω_+ and ω_- are given by [9]

$$r = \frac{f_B}{2f_K^2} - \frac{f_B}{f_K^2}\frac{p_B.(p_2 - p_1)}{(p_B - p_1 - p_2)^2 - m_B^2}$$
$$+ \frac{2gf_{B_s^*}}{f_K^2}\sqrt{\frac{m_B}{m_{B_s^*}}}\frac{(p_B - p_1).p_1}{(p_B - p_1)^2 - m_{B_s^*}^2}$$
$$- \frac{4g^2 f_B}{f_K^2}\frac{m_B m_{B_s^*}}{(p_B - p_1 - p_2)^2 - m_B^2}$$
$$B \to \bar{D}^* K^+ \bar{K}^0$$
$$\times \frac{p_1.p_2 - (p_B - p_1).p_1 (p_B - p_1).p_2 / m_{B_s^*}^2}{(p_B - p_1)^2 - m_{B_s^*}^2}.$$

$$\omega_+ = -\frac{g}{f_K^2}\frac{f_{B_s^*} m_{B_s^*}\sqrt{m_B m_{B_s^*}}}{(p_B - p_1)^2 - m_{B_s^*}^2}\left[1 - \frac{(p_B - p_1).p_1}{m_{B_s^*}^2}\right] + \frac{f_B}{2f_K^2},$$

$$\omega_- = -\frac{g}{f_K^2}\frac{f_{B_s^*} m_{B_s^*}\sqrt{m_B m_{B_s^*}}}{(p_B - p_1)^2 - m_{B_s^*}^2}\left[1 + \frac{(p_B - p_1).p_1}{m_{B_s^*}^2}\right]. \qquad (4)$$

The other two-body matrix element can be related to the kaon matrix element of the weak interaction current

$$\left\langle K^+(p_1)\bar{K}^0(p_2)\right|(u\bar{d})_{V-A}\left|0\right\rangle = (p_2 - p_1)_\mu F^{K^+\bar{K}^0}. \qquad (5)$$

Here the momentum dependence of the weak form factor $F^{K^+\bar{K}^0}$ is parametrized as

$$F^{K^+\bar{K}^0}(q^2) = \frac{F^{K^+\bar{K}^0}(0)}{1 - q^2/\Lambda_\chi^2 + i\Gamma_*/\Lambda_\chi}, \qquad (6)$$

where $q^2 = s = (p_B - p_{D^*})^2$ and $\Gamma_* = 0.20$ Gev, and $\Lambda_\chi = 0.83$ Gev is the chiral-symmetry breaking scale. We need also the $B \to D^*$ form factor and the $0 \to D^*$ decay constant, which are defined as follows [14]

$$\left\langle D^*(\varepsilon_3, p_3)\right|(c\bar{b})_{V-A}\left|B\right\rangle =$$
$$-i(\varepsilon_{3\mu} - \frac{\varepsilon_3.q}{q^2}q_\mu)(m_B + m_{D^*})A_1(q^2)$$
$$+ i\left((p_B + p_3)_\mu - \frac{m_B - m_{D^*}}{q^2}q_\mu\right)$$
$$\times(\varepsilon_3.q)\frac{A_2(q^2)}{m_B + m_{D^*}} - i\frac{2m_{D^*}(\varepsilon_3.q)}{q^2}q_\mu A_0(q^2)$$
$$\left\langle 0\right|V_\mu\left|D^*(\varepsilon_3, p_3)\right\rangle = f_{D^*} m_{D^*}\varepsilon_3. \qquad (7)$$

Now we can obtain the decay amplitudes as

$$M(B^+ \to K^+(p_1)\bar{K}^0(p_2)\bar{D}^{*0}(\varepsilon_3, p_3)) =$$
$$i\frac{G_F}{\sqrt{2}}f_{D^*}m_{D^*}a_2 V_{cb}^* V_{ud}(r\varepsilon_3.(p_B - p_1 - p_2)$$
$$+ \omega_+\varepsilon_3.(p_2 + p_1) + \omega_-\varepsilon_3.(p_2 - p_1))$$
$$-i\frac{G_F}{\sqrt{2}}F^{K^+\bar{K}^0}a_1 V_{cb}^* V_{ud}(\varepsilon_3.(p_2 - p_1)$$
$$-\frac{\varepsilon_3.q}{q^2}q.(p_2 - p_1))(m_B + m_{D^*})A_1(q^2)$$
$$-((p_B + p_3).(p_2 - p_1)$$
$$-\frac{m_B - m_{D^*}}{q^2}q.(p_2 - p_1))(\varepsilon_3.q)\frac{A_2(q^2)}{m_B + m_{D^*}}$$
$$+\frac{2m_{D^*}(\varepsilon_3.q)}{q^2}q.(p_2 - p_1)A_0(q^2), \qquad (8)$$

and

$$M(B^0 \to K^+(p_1)\bar{K}^0(p_2)D^{*-}(\varepsilon_3, p_3)) =$$
$$-i\frac{G_F}{\sqrt{2}}F^{K^+\bar{K}^0}a_1 V_{cb}^* V_{ud}(\varepsilon_3.(p_2 - p_1)$$
$$-\frac{\varepsilon_3.q}{q^2}q.(p_2 - p_1))(m_B + m_{D^*})A_1(q^2)$$
$$-((p_B + p_3).(p_2 - p_1)$$
$$-\frac{m_B - m_{D^*}}{q^2}q.(p_2 - p_1))(\varepsilon_3.q)\frac{A_2(q^2)}{m_B + m_{D^*}}$$
$$+\frac{2m_{D^*}(\varepsilon_3.q)}{q^2}q.(p_2 - p_1)A_0(q^2), \qquad (9)$$

where

$$a_1 = c_1 + \frac{c_2}{3},$$
$$a_2 = c_2 + \frac{c_1}{3},$$
$$q.(p_2 - p_1) = (p_2 + p_1).(p_2 - p_1) = m_{K^+}^2 - m_{\bar{K}^0}^2 \cong 0, \qquad (10)$$

and under the Lorentz condition $\varepsilon_3.p_3 = 0$. The $\bar{D}^*$meson polarization vectors become

$$\varepsilon_3^{(\lambda=0)} = (|\vec{p}_3|, 0, 0, p_3^0)/m_3,$$
$$\varepsilon_3^{(\lambda=\pm1)} = m(0, 1, \pm i, 0)/\sqrt{2}. \qquad (11)$$

Consider the decay of B meson into three particles of masses m_1, m_2 and m_3. Let's denote their 4-momenta by p_B, p_1, p_2 and p_3, respectively. Energy-momentum conservation is expressed by

$$p_B = p_1 + p_2 + p_3, \qquad (12)$$

Define the following invariants

$$s_{12} = (p_1 + p_2)^2 = (p_B - p_3)^2,$$
$$s_{23} = (p_2 + p_3)^2 = (p_B - p_1)^2,$$

$$s_{31}=(p_3+p_1)^2=(p_B-p_2)^2 \ . \quad (13)$$

The three invariants s_{12}, s_{23} and s_{31} are not independent. it follows from their definitions together with 4-momentum conservation that

$$s_{12}+s_{23}+s_{31}=m_B^2+m_1^2+m_2^2+m_3^2 \ . \quad (14)$$

We take $s_{12}=s$ and $s_{23}=t$, so we have $s_{31}=m_B^2+m_1^2+m_2^2+m_3^2-s-t$. In the center of mass of $K^+(p_1)$ and $\bar{K}^0(p_2)$, according to figure 2, we find

$$|\vec{p}_1|=|\vec{p}_2|=\frac{1}{2}\sqrt{s-4m_1^2},$$

$$p_1^0=p_2^0=\frac{1}{2}\sqrt{s},$$

$$|\vec{p}_3|=p_3^3=\frac{1}{2\sqrt{s}}\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2},$$

$$p_3^0=\frac{1}{2\sqrt{s}}(m_B^2-m_3^2-s)^2,$$

$$|\vec{\varepsilon}_3|=\frac{1}{2m_3}(m_B^2-m_3^2-s)^2,$$

$$\varepsilon_3^0=\frac{1}{2m_3\sqrt{s}}\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2} \ , \quad (15)$$

and the cosine of the helicity angle θ between the direction of $\vec{p}_2$ and that of $\vec{p}_3$ reads

$$\cos\theta=\frac{1}{4|\vec{p}_2||\vec{p}_3|}(m_B^2+m_3^2+2m_2^2-s-2t) \ . \quad (16)$$

With these definitions, we obtain multiplying of the 4-momentum as

$$\varepsilon_3.(p_2+p_1)=2p_1^0\varepsilon_3^0,$$

$$\varepsilon_3.(p_2-p_1)=2|\vec{\varepsilon}_3||\vec{p}_1|\cos\theta \ . \quad (17)$$

Finally the decay amplitudes can be derived as

$$M(B^+\to K^+(p_1)\bar{K}^0(p_2)\bar{D}^{*0}(\varepsilon_3,p_3))=$$
$$i\frac{G_F}{\sqrt{2}}f_{D^*}a_2V_{cb}^*V_{ud}(\omega_+\sqrt{(m_B^2-m_3^2-s)^2-4sm_3^2}$$
$$+\omega_-(m_B^2-m_3^2-s)\sqrt{1-4m_1^2/s}\cos\theta)$$
$$-i\frac{G_F}{2\sqrt{2}m_{D^*}}F^{K^+\bar{K}^0}a_1V_{cb}^*V_{ud}((m_B+m_{D^*})$$
$$\times(m_B^2-m_3^2-s)\sqrt{1-4m_1^2/s}\cos\theta A_1(q^2)$$
$$-\sqrt{1-4m_1^2/s}\left[(m_B^2-m_3^2-s)^2-4sm_3^2\right]\cos\theta$$
$$\times\frac{A_2(q^2)}{m_B+m_{D^*}} \ , \quad (18)$$

and

$$M(B^+\to K^+(p_1)\bar{K}^0(p_2)\bar{D}^{*0}(\varepsilon_3,p_3))=$$
$$-i\frac{G_F}{2\sqrt{2}m_{D^*}}F^{K^+\bar{K}^0}a_1V_{cb}^*V_{ud}((m_B+m_{D^*})$$
$$\times(m_B^2-m_3^2-s)\sqrt{1-4m_1^2/s}\cos\theta A_1(q^2)$$
$$-\sqrt{1-4m_1^2/s}\left[(m_B^2-m_3^2-s)^2-4sm_3^2\right]\cos\theta$$
$$\times\frac{A_2(q^2)}{m_B+m_{D^*}} \ . \quad (19)$$

The decay width of a three-body process is given by [15]

$$\Gamma(B\to D^*KK)=\frac{1}{(2\pi)^3}\frac{1}{32m_B^3}$$
$$\int_{s_{\min}}^{s_{\max}}\int_{t_{\min}}^{t_{\max}}|M(B\to D^*KK)|^2\,ds dt \ , \quad (20)$$

where

$$t_{\min,\max}(s)=m_1^2+m_3^2$$
$$-\frac{1}{2s}((m_B^2-m_3^2-s)(s-m_2^2+m_1^2)$$
$$\mp\sqrt{\lambda(s,m_B^2,m_3^2)}\sqrt{\lambda(s,m_1^2,m_2^2)}).$$

$$s_{\min}=(m_1+m_2)^2, s_{\max}=(m_B-m_3)^2 \ , \quad (21)$$

where

$$\lambda(x,y,z)=x^2+y^2+z^2-2(xy+yz+zx).$$

2.2 Our Assumption

We work in the rest frame of the B meson, when B decays to final states mesons.The D^* meson due to heavy mass has a small momentum along the z axis and makes $\pi-\theta$ degrees with momenta of the K mesons. Momentums of the B and final states mesons are given by

$$p_B=(m_B,0,0,0),\ p_{D^*}=(p_3^0,0,0,p_{z3}),$$
$$p_{K^+}=(p_1^0,0,p_{y1},-p_{z1}),\ p_{\bar{K}^0}=(p_2^0,0,-p_{y2},-p_{z2}), \quad (22)$$

where $|\vec{p}_1|=|\vec{p}_2|, p_1^0=p_2^0=(m_B-m_{D^*})/2$ and $p_{3z}=2|\vec{p}_K|\cos\theta$. With this assumption we are able to obtain all of the momenta for each arbitrary θ, in which case the amplitudes become constant. After simplifying the Eqs. (8) and (9), we then find the simple amplitudes

$$M(B^+\to\bar{D}^{*0}(2007)K^+\bar{K}^0)$$
$$=i\sqrt{2}G_FV_{cb}^*V_{ud}a_2f_{D^*}\omega_+(p_{z3}p_1^0+p_{z1}p_3^0)$$
$$-iG_FV_{cb}^*V_{ud}a_1F^{K^+\bar{K}^0}A_1^{BD^*}(q^2)(m_B+m_{D^*})p_{y1} \quad (23)$$

and

$$M(B^0\to D^{*-}(2010)K^+\bar{K}^0)$$
$$=-iG_FV_{cb}^*V_{ud}a_1F^{K^+\bar{K}^0}A_1^{BD^*}(q^2)(m_B+m_{D^*})p_{y1} \quad (24)$$

The decay rate of $B^+\to\bar{D}^{*0}(2007)K^+\bar{K}^0$ and $B^0\to D^{*-}(2010)K^+\bar{K}^0$ decays is then given by

$$\Gamma(B\to D^*KK)=\frac{1}{(2\pi)^3}\frac{1}{32m_B^3}st|M(B\to D^*KK)|^2 \quad (25)$$

where

$$s=(p_B-p_{D^*})^2=m_B^2+m_{D^*}^2-2m_Bm_{D^*},$$

Table 1. Wilson coefficients c_i in the NDR scheme at three different choices of the renormalization scale μ .

NLO	$\mu = m_b / 2$	$\mu = m_b$	$\mu = 2m_b$
c_1	1.137	1.081	1.045
c_2	–0.295	–0.190	–0.113

Table 2. Branching ratios of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ (in units of 10^{-3}) and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ (in units of 10^{-4}) decays by using the different angles between the K mesons according to our assumptions at $\mu = 2m_b$ scale.

2θ	$B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$	$B^0 \to D^*(2010)^- K^+ \bar{K}^0$
0°	0.000	0.000
30°	0.049 ± 0.004	0.226 ± 0.017
60°	0.128 ± 0.009	0.449 ± 0.036
90°	0.159 ± 0.011	0.790 ± 0.059
120°	0.207 ± 0.015	1.088 ± 0.079
150°	0.270 ± 0.019	1.410 ± 0.102
180°	0.318 ± 0.023	1.726 ± 0.125
Exp. [1,2]	< 1.060	< 4.700

Table 3. Branching ratios of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ (in units of 10^{-3}) and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ (in units of 10^{-4}) decays.

Mode	$B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$	$B^0 \to D^*(2010)^- K^+ \bar{K}^0$
$\mu = m_b / 2$	0.188 ± 0.013	1.603 ± 0.116
$\mu = m_b$	0.250 ± 0.018	1.659 ± 0.120
$\mu = 2m_b$	0318 ± 0.023	1.726 ± 0.125
all degrees	4.862 ± 0.352	49.212 ± 3.559
up to 105°	0.475 ± 0.034	4.677 ± 0.338
Exp. [1,2]	< 1.060	< 4.700

$$t = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B p_K^0 . \quad (26)$$

3.Numerical Results

The Wilson coefficients c_i have been calculated in different schemes. In this paper, we use consistently the naïve dimensional regularization(NDR) scheme. The values of c_i at the scales $\mu = m_b / 2, \mu = m_b$ and $\mu = 2m_b$ next to leading order (NLO) are shown in table 1 [16].

The parameter g in the form factors, determined from the $D^* \to D\pi$ decay, is [9]

$$g = 0.3 \pm 0.1. \quad (27)$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix as [17]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \quad (28)$$

The elements of the CKM matrix can be parameterized by three mixing angles A, λ, ρ and a CP-violating phase η

$$V_{ud} = 1 - \lambda^2 / 2,\ V_{us} = \lambda,\ V_{ub} = A\lambda^3(\rho - i\eta),$$
$$V_{cd} = -\lambda,\ V_{cs} = 1 - \lambda^2 / 2,\ V_{cb} = A\lambda^2,$$
$$V_{td} = A\lambda^3(1 - \rho - i\eta),\ V_{ts} = -A\lambda^2,\ V_{tb} = 1 . \quad (29)$$

The results for the Wolfenstein parameters are

$$\lambda = 0.2257 \pm 0.001,\ A = 0.814 \pm 0.022$$
$$\bar{\rho} = 0.135 \pm 0.023,\ \bar{\eta} = 0.349 \pm 0.016. \quad (30)$$

Then we obtain

$$|V_{ud}| = 0.97452 \pm 0.00023,$$
$$|V_{us}| = 0.22570 \pm 0.00100,$$
$$|V_{ub}| = 0.00350 \pm 0.00035,$$
$$|V_{cd}| = 0.22570 \pm 0.00100,$$
$$|V_{cs}| = 0.97452 \pm 0.00023,$$
$$|V_{cb}| = 0.04147 \pm 0.00150,$$
$$|V_{td}| = 0.00873 \pm 0.00021,$$
$$|V_{ts}| = 0.04147 \pm 0.00150,$$
$$|V_{tb}| = 1. \quad (31)$$

The meson masses and decay constants needed in our calculations are taken as (in units of MeV) [17]

$$m_B = 5279.25 \pm 0.17,$$
$$m_{B_s^*} = 5415.40 \pm 2.25,$$
$$m_{D^*} = 2006.98 \pm 0.15,$$
$$m_K = 493.677 \pm 0.016,$$
$$f_B = 176, f_{B_s^*} = 220, f_{D^*} = 230, f_K = 160. \quad (32)$$

The values of the form factors $A_1^{BD^*}$ and $F^{K^+\bar{K}^0}$ are given by

$$A_1^{BD^*} = 1.13 \; F^{K^+\bar{K}^0} = 0.067 \qquad (33)$$

Using the parameters relevant for the $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays, we calculate the branching ratios of these decays in two different ways, which are shown in tables 2 and 3. Because the integral is taken over all values of θ, which is the angle between K mesons, the results obtained by using the Dalitz plot model are larger than the experimental value. Assuming a static D^*, as can be achieved from table 2, for $\mu = 2m_b$ and $\theta = 180^\circ$, we obtain $BR(B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0) = 0.318 \times 10^{-3}$ and $BR(B^0 \to D^*(2010)^- K^+ \bar{K}^0) = 1.726 \times 10^{-4}$, which are the best answers close to the experimental values.

4. Conclusion

In this work, we have calculated the branching ratios of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ the and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays by using the Dalitz plot analysis. According to QCD factorization approach, we have obtained $BR(B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0) = 4.862 \times 10^{-3}$ and $BR(B^0 \to D^*(2010)^- K^+ \bar{K}^0) = 49.212 \times 10^{-4}$, while the experimental results of them are less than 1.060×10^{-3} and 4.700×10^{-4}, respectively. We have assumed that, in the $B \to D^* KK$ decays, because the mass of the D^* is too heavy against the K mesons, the momentum of the D^* can have a small amount. Namely, the D^* carries a small momentum. It seems that in the general form of Dalitz plot technique, because the integral is taken over all values of θ, the results obtained by using this model are larger than the experimental value. We have found that in the probability summation of the amplitudes, the momentum of the D^*, instead of starting from its maximum value, starts from a state in which two light mesons make an angle of 105 degrees. By using our findings, we have calculated the branching ratios of $B^+ \to \bar{D}^*(2007)^0 K^+ \bar{K}^0$ and $B^0 \to D^*(2010)^- K^+ \bar{K}^0$ decays and obtained 0.475×10^{-3} and 4.677×10^{-4} respectively. In addition, we have estimated the branching ratios for different angles between the two light mesons and got that when the angle between them is 180°, in fact they decay back to back, andthese values become 0.318×10^{-3} and 1.726×10^{-4} which are the best answers close to the experimental values. The branching ratios obtained by applying our assumption are compatible with the experimental results.

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