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Research note

Electrostatic compressive and rarefactive dust ion-acoustic solitons in four component quantum plasma

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Abstract

The propagation of nonlinear quantum dust ion-acoustic (QDIA) solitary waves in a unmagnetized quantum plasma whose constituents are inertialess quantum electrons and positrons, classical cold ions and stationary negative dust grains are studied by deriving the Korteweg–de Vries (KdV) equation under the reductive perturbation method. Quantum Hydrodynamic (QHD) equations are used to take into account the quantum diffraction in quantum statistics corrections. It is shown that depending on some critical values of the dust density (d) which is function of quantum diffraction parameter (H), both rarefactive and compressive type of solitons can exist in the model plasma. Further, the amplitude and width of both solitons increase as d increases. Moreover, it is pointed out that an increase in quantum diffraction parameter, decreases the width of compressive soliton but increases the width of rarefactive soliton, and the amplitude of both solitons is independent of H. The present investigation could be useful for researches on astrophysical plasmas as well as for ultra small micro- and nano- electronic devices.

Keywords: Bohm potential, compressive solitons, dust density, KdV equation, quantum plasma, rarefactive solitons

1. Introduction

Quantum plasma is characterized by high-plasma particle number densities and low-temperatures, in contrast to classical plasma which has high-temperatures and low particle number densities. These plasmas obey the Fermi-Dirac distribution leading to the Fermi pressure and new forces arising due to the Bohm potential [1] play a vital role. The de Broglie wavelength of plasma particles is defined as $\lambda_{Bj} = h/(2\pi m_j V_{Tj})$ with m_j and V_{Tj} being the mass and thermal velocity of jth species. Furthermore, quantum effects associated with strong density correlation start playing a significant role when de-Broglie wavelength is larger than average interparticle distance ($d = n^{\frac{-1}{3}}$), i.e., $n\lambda_B \geq 1$.

Three well known mathematical formulations to describe the dynamics of quantum plasmas are Schrodinger-Poisson model, Wigner-Poisson model and quantum hydrodynamic (QHD) model. QHD model is a useful approximation to study short scale collective phenomena such as waves, instabilities, and nonlinear structures, etc. in quantum plasmas. This model generalizes the fluid model for plasmas and takes into account the macroscopic variables only, i.e., density, fluid velocity, stress tensor and electrostatic potential. These models have been discussed in detail in Manfredi and Haas [1] and Haas [2].

Quantum plasmas are common in different environments, e.g. in superdense astrophysical bodies [3] (i.e. interior of Jupiter, massive white dwarfs and neutron stars), in intense laser-solid density plasma experiments [4, 5, 6], and in ultrasmall electronic devices (e.g. in microelectronics, semiconductor devices [7], quantum dots, nanowires [8], carbon nanotubes [9], quantum diodes [10], ultracold plasmas [11], and microplasmas [12]).

Quantum plasmas have received great attention in recent decades mainly due to the wide applications mentioned above [13-19]. Many of these researches in quantum plasmas are dedicated to electron-positron-ion (e-p-i) quantum plasmas, because it is believed that enormously high density e-p-i quantum plasmas may exist in astrophysical environments as well as in laser-solid matter interaction plasmas [20].

Many authors have studied the effects of quantum diffraction and Fermi pressure on linear and nonlinear electrostatic waves in dense e-p-i plasmas. It is found

that Bohm potential leads to the wave dispersion due to quantum correlation of density fluctuations associated with wave-like nature of the charge carriers. For instance, Khan and Haque [21] investigated electrostatic nonlinear structures in dissipative e- p-i plasma and observed that the quantum diffraction parameter decreased the width of the ion acoustic solitary wave. Ali et al. [22] studied linear and nonlinear ion- acoustic waves in an unmagnetized e-p-i quantum plasma and found that the solitary waves in this plasma behaved quite differently than that of ordinary e-i plasma. In particular, they found that an increase in positron concentration decreased the amplitude and width of solitary waves; and the ratio of positron Fermi temperature to electron Fermi temperature increased the amplitude and width of the solitary waves.

Moreover, in some quantum plasmas in addition to electrons, ions, and positrons, an additional component of dust particles can also exist; for instance, microelectronic devices and metallic nanostructures, space and astrophysical plasmas, plasma coating, tokamak edges, and interplanetary spaces are usually doped or contaminated by the presence of highly charged dust impurities. The presence of dust particles changes the equilibrium condition and introduces new types of ion acoustic waves. So it is therefore interesting and worthwhile to examine the effect of dust grain density as well as quantum diffraction parameter on the quantum dust ion acoustic (QDIA) waves in e-p-i quantum plasma. To the best of our knowledge, no investigation for an e-p-i-dust plasma has been made of the nonlinear propagation of electrostatic waves.

Therefore, in this study, QDIAS–waves has been investigated in an unmagnetized, collisionless four component quantum plasma containing inertialess quantum electrons and positrons, classical cold ions and stationary negative dust grains. It has also been investigated that how the quantum corrections and the presence of dust density modify the wave structures in quantum e-p-i-dust plasma. In the present study, first the basic set of equations for QDIAS waves in e- p- i-dust plasma is presented. Derivation of KdV equation is given using reductive perturbation method [23]. Then the solutions of KdV are investigated. Results of this study would be helpful for understanding the structures that may occur in space plasmas and to find the properties of dense astrophysical (i.e., white dwarfs and neutron stars).

2. Basic equations

Here a one-dimensional, unmagnetized collisionless four-component quantum dusty plasma consisting of inertialess quantum electrons and positrons, classical cold ions and stationary negative dust grains to study the nonlinear propagation of ion acoustic solitary waves. It is also assumed that the plasma particles for a one dimensional zero-temperature Fermi gas obey the pressure law [24].

$$P_j = \frac{m_j V_{Fj}^2 n_j^3}{3 n_{j0}^2}, \tag{1}$$

where j equals e for electrons and p for positrons. m_j is the mass, $V_{Fj} = \left(2K_B T_{Fj}/m_j\right)^{1/2}$ is the Fermi speed, K_B is the Boltzmann constant, $T_{Fj} = \frac{\hbar^2}{2m_j K_B}(3\pi^2)^{\frac{2}{3}} n_j^{\frac{2}{3}}$ is the Fermi temperature and $\hbar$ is the Planck constant. Furthermore, n_j is the number density with its equilibrium value n_{j0}. At equilibrium, the charge neutrality condition is $\alpha + d = 1 + p$ where $d = \frac{n_{d0} z_{d0}}{n_{i0}}$, $p = \frac{n_{p0}}{n_{i0}}$, $\alpha = \frac{n_{e0}}{n_{i0}}$ and z_{d0} is the dust charging state. The normalized basic set of equations for DIAW are given below,

$$\partial_t u + u\partial_x u = -\partial_x \phi, \tag{2}$$

$$\partial_t n_i + \partial_x \left(n_i u\right) = 0, \tag{3}$$

$$\partial_x \phi - \frac{1}{n_e}\partial_x n_e + \delta \frac{H^2}{2}\partial_x \left[\frac{\partial_x^2 \sqrt{n_e}}{\sqrt{n_e}}\right] = 0, \tag{4}$$

$$-\partial_x \phi - \frac{\sigma}{n_p}\partial_x n_p + \delta \frac{H^2}{2}\partial_x \left[\frac{\partial_x^2 \sqrt{n_p}}{\sqrt{n_p}}\right] = 0, \tag{5}$$

$$\partial_x^2 \phi = \mu_d \left(1-d\right) n_e + \left(1-\mu_d\right)\left(1-d\right) n_p - n_i + d. \tag{6}$$

The number density, n_j ($j = e, i, p, d$), of the jth species are normalized by their unperturbed density n_{j0}, the electrostatic wave potential φ is normalized by $K_B T_{Fe}/e$, and the fluid ion velocity u is normalized by quantum ion-acoustic speed $c_q = \left(K_B T_{Fe}/m_i\right)^{1/2}$. The space and time coordinates (x and t) are normalized, respectively, by the quantum Debye length $\lambda_D = \left(K_B T_{Fe}/4\pi e^2 n_{i0}\right)^{1/2}$ and the ion plasma period $\omega_{pi}^{-1} = \left(4\pi n_{i0} e^2/m_i\right)^{-1/2}$. Further, $m_e = m_p = m$, $\sigma = T_{Fp}/T_{Fe}$, $\mu_d = \alpha/1-d = 1 + p/1-d$, $\delta = n_{i0}/n_{e0}$. The nondimensional quantum diffraction parameter H is defined as $H = \hbar\omega_{pe}/2K_B T_{Fe}$ where $\omega_{pe} = \left(4\pi n_{e0} e^2/m_e\right)^{\frac{1}{2}}$.

Quantum diffraction effects appear in this system through third term of eqs. (4) and (5), and quantum statistical effects have been shown through the second term of eqs. (4) and (5).

3. KdV equation

Now, KdV equation is derived from eqs. (2)-(6) by reductive perturbation technique. The stretched coordinates $\xi = \varepsilon^{\frac{1}{2}}\left(x - \lambda t\right)$ and $\tau = \varepsilon^{\frac{3}{2}} t$ are introduced where ε is the small nonzero parameter proportional to the amplitude of the perturbation and λ is the linear wave velocity. The perturbed quantities can be expanded as

$$n_j = 1 + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \cdots,$$
$$u = 0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots, \tag{7}$$
$$\phi = 0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots \quad .$$

Substituting the expressions from eq. (7) along with stretching coordinates into eqs. (2-6) and collecting the terms in different orders of ε. The lowest orders in ε, i.e., $O(\varepsilon)$ and $O(\varepsilon^{\frac{3}{2}})$, leads to the following relations:

$$n_{e1} = \varphi_1, \tag{8}$$
$$n_{p1} = -\frac{\varphi_1}{\sigma}, \tag{9}$$
$$n_{i1} = (1-d)\left[\mu_d - (1-\mu_d)/\sigma\right]\varphi_1, \tag{10}$$
$$u_1 = \frac{\varphi_1}{\lambda}, \tag{11}$$
$$\lambda = \left[(1-d)\left\{\mu_d - (1-\mu_d)/\sigma\right\}\right]^{\frac{-1}{2}}. \tag{12}$$

Similarly $O(\varepsilon^2)$ and $O(\varepsilon^{\frac{5}{2}})$ comes out to be:

$$-\lambda\partial_\xi n_{i2} + \partial_\xi u_2 = -\partial_\tau n_{i1} - \partial_\xi (n_{i1}u_1), \tag{13}$$
$$\partial_\xi \phi_2 - \lambda\partial_\xi u_2 = -\partial_\tau u_1 - u_1\partial_\xi u_1, \tag{14}$$
$$n_{e2} = \varphi_2 - \frac{\varphi_1^2}{2} + \frac{H^2\delta}{4}\partial_\xi^2\varphi_1, \tag{15}$$
$$n_{p2} = -\frac{\varphi_2}{\sigma} - \frac{\varphi_1^2}{2\sigma^2} - \frac{H^2\delta}{4\sigma^2}\partial_\xi^2\varphi_1, \tag{16}$$
$$\partial_\xi^2\varphi_1 = (1-d)\mu_d n_{e2} + (1-d)(1-\mu_d)n_{p2} - n_{i2} \ . \tag{17}$$

Eliminating the second order quantities from the above equations using the first order relations, the following Korteweg-de Vries (KdV) equation is obtained:

$$\partial_\tau\Phi + A\Phi\partial_\xi\Phi + B\partial_\xi^3\Phi = 0, \tag{18}$$

which $\Phi = \varphi_1$ and the coefficients of nonlinearity (A) and dispersion (B) are defined as

$$A = \frac{3}{2\lambda} + \frac{\lambda^3}{2}(1-d)\left[\mu_d + (1-\mu_d)/\sigma^2\right], \tag{19}$$
$$B = \frac{\lambda^3}{2}\left[1 - \frac{H^2}{4}\left[1 - \frac{(1-\mu_d)}{\mu_d\sigma^2}\right]\right], \tag{20}$$

Note that the primary effect of quantum diffraction of electrons and positrons is the wave dispersion as it does not contribute in the nonlinearity coefficient A. To find the steady state solution of eq. (18), independent variables of ξ and τ are being transferred into one variable $\eta = \xi - u_0\tau$ where u_0 is the normalized constant speed of the wave frame. Applying the boundary conditions as $\eta \to \pm\infty$; $\Phi \to 0$, $d\Phi/d\eta \to 0$, and $d^2\Phi/d\eta^2 \to 0$ the possible stationary solution of eq. (18) is obtained as:

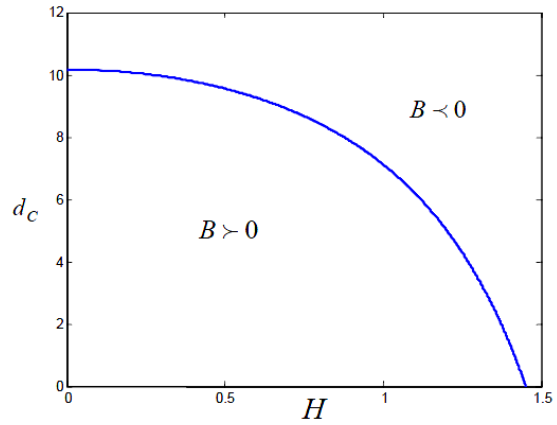


Figure 1. Critical values of d against H. $p = 9.17$ for $\sigma = 1$.

$$\Phi(\eta) = \Phi_m \sec h^2\left(\frac{\eta}{w}\right), \tag{21}$$

where $\Phi_m = 3u_0/A$ and $w = \sqrt{\frac{4B}{u_0}}$ are the amplitude and width of the soliton respectively.

4. Compressive and rarefactive solitons

Solitary wave structure is found due to a delicate balance between the dispersive effect and nonlinear effect. Relative strength of these two effects determines the characteristic of such solitary wave structure. Coefficients A and B thus, play a crucial role in determining the solitary wave structure. From eqs. (19) and (20), it is found that these coefficients get modified by dust density and quantum effects. So it is important to study the dependence of these coefficients on dust density (d) and quantum diffraction parameter H. The coefficient A is independent of H but depends on d and the coefficient B depends interestingly on d and H. Calculations show that the value of B vanishes at some value of d (say d_C), where d_C is a function of σ, p and H. It means that for a certain value of this parameter, there exist a critical value for dust density in which KdV soliton disappears. For $\sigma = 1$, this critical value is $d_C = 1 + p(1 - H^2/4 - H^2)$. The continuous curve in figure 1 shows the variation of d_C with H, which is the solution of B=0. From eq. (21) it is found that in all physically acceptable situation with $B > 0$ (below) and $u_0 > 0$, only compressive solitary wave structure is obtained and it is clear that no soliton solution is possible for $B < 0$ (above) with velocity $u_0 > 0$. However, for $B < 0$ formation of solitary wave structure is possible only for $u_0 < 0$. In this case, rarefactive soliton is obtained. Therefore, from the sign of B, one can show that only compressive KdV solitons can propagate in the plasma for $d < d_C$ ($B > 0$) which is below the continuous curve, and only rarefactive KdV solitons can be launched when $d > d_C$ ($B < 0$) which is obtained

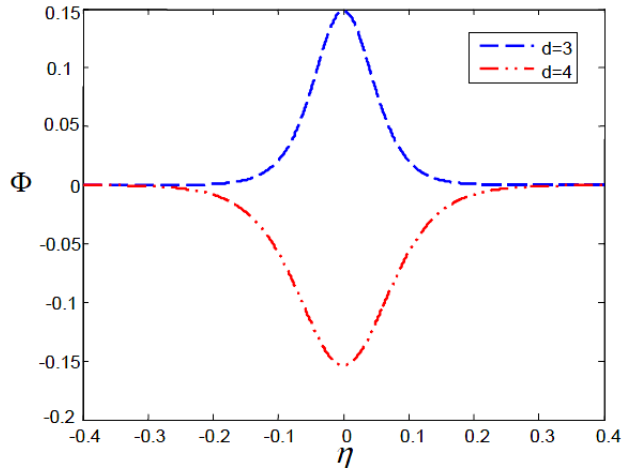


Figure 2. QDIA compressive and rarefactive solitons for different values of *d*, with $H=1.3$, $|u_0|=0.3$, $\sigma=1$ and $p=9.17$.

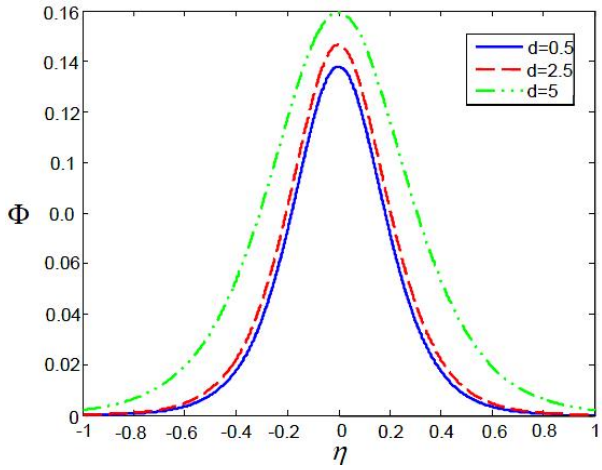


Figure 3. QDIA compressive solitons for different values of *d*, with $H=0.8$, $u_0=0.3$, $\sigma=1$ and $p=9.17$.

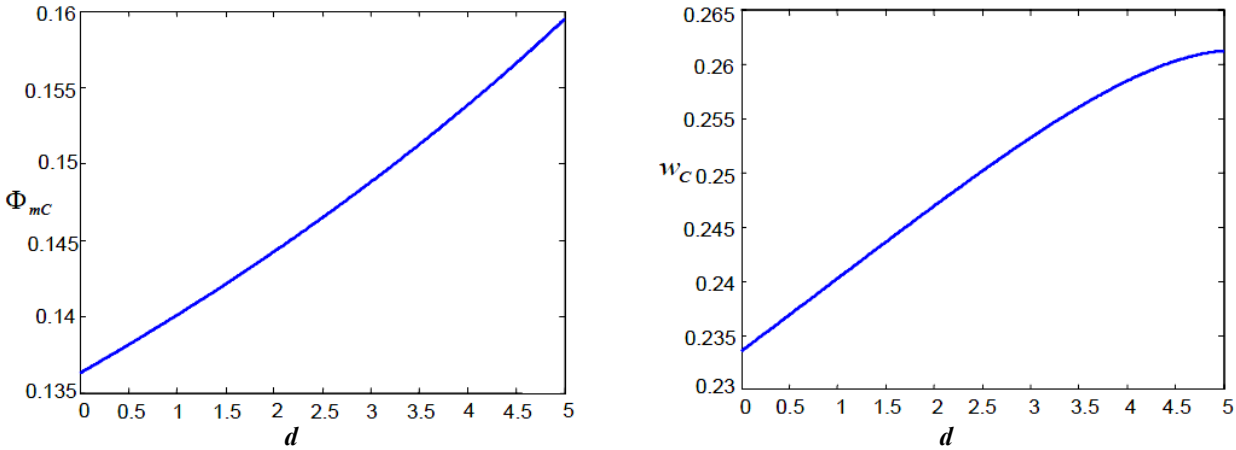


Figure 4. (a) Dependence of QDIA compressive soliton amplitude (Φ_{mC}) on *d*. (b) Dependence of QDIA compressive soliton width (w_C) on *d*.Other parameters are $H=0.8$ ، $u_0=0.3$ ، $\sigma=1$ and $p=9.17$.

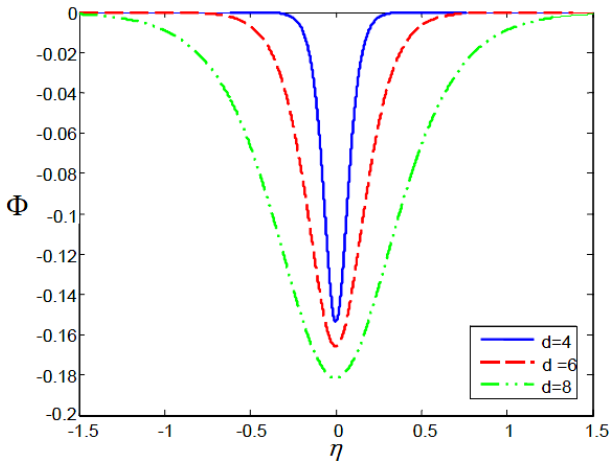


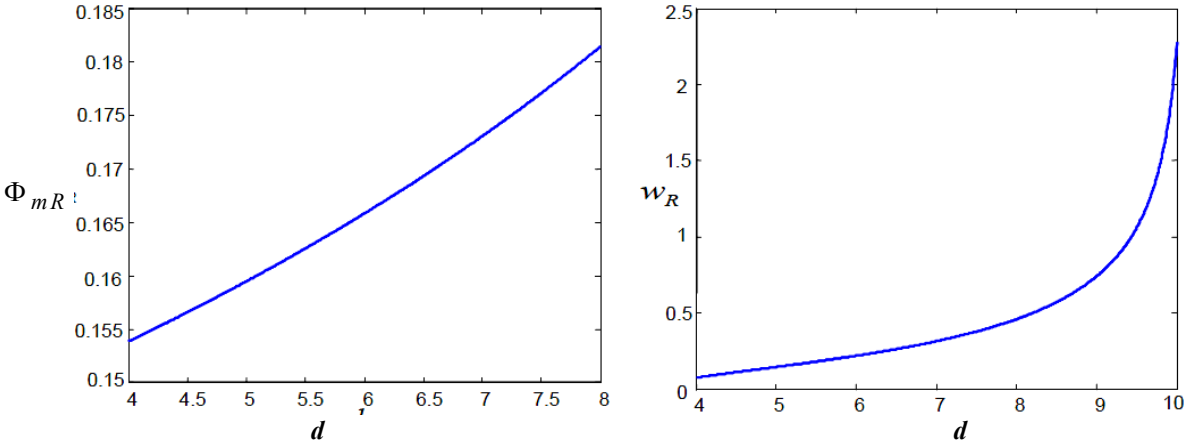
Figure 5. QDIA rarefactive solitons for different values of *d*, with $H=1.3$, $u_0=-0.3$, $\sigma=1$ and $p=9.17$.

above the continuous curve in figure 1.

Figure 1 shows that for a plasma with *H*=1.3, the amplitude of KdV solitons would be positive for small values of d (for instance *d*=3), while for larger ones ($d\geq 4$), there are negative potential solitons ($d>d_C$).

Figure 2 shows the effect of dust density on soliton with constant *H* (=1.3). Moreover dust density affects the structure of compressive and rarefactive solitons (figures 3 and 5). Figures 4 (a) and 4 (b) respectively show that the amplitude of compressive soliton and its width that increases with the increase of *d*. Also, from figure 6, it is clear that the amplitude and width of rarefactive soliton significantly increase as *d* increases.

Figures 7 and 8 show the effect of *H* on soliton with constant *d* (=4). From figure 1, it is clear that for a plasma with *d*=4, compressive soliton can exist for small values of *H* (for instance $H<1.268$) and rarefactive soliton can exist for larger ones ($H>1.268$).
Moreover, quantum diffraction parameter (*H*) affects the structure of compressive and rarefactive solitons (Figures 7 and 9). It is shown that the amplitude of both solitons does not depend on the quantum diffraction parameter *H*, but the width of compressive soliton decreases significantly (Figure 8), and the width of rarefactive soliton increases (Figure 10) as *H* increases.



(a) (b)

Figure 6. (a) Dependence of QDIA rarefactive soliton amplitude (Φ_{mR}) on d.(b) Dependence of QDIA rarefactive soliton width (w_R) on d. Other parameters are $H=1.3$, $u_0=-0.3$, $\sigma=1$ and $p=9.17$.

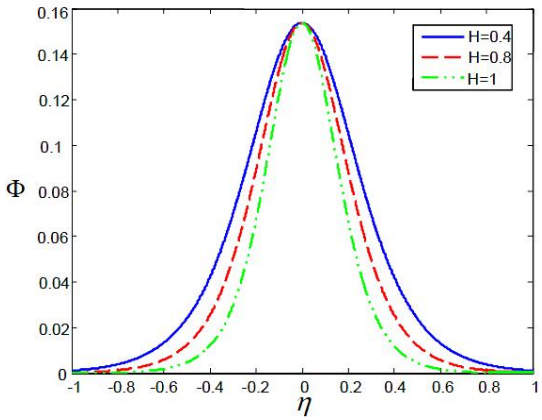


Figure 7. QDIA compressive solitons for different values of H, with $u_0=0.3$, $p=9.17$, $\sigma=1$, $d=4$.

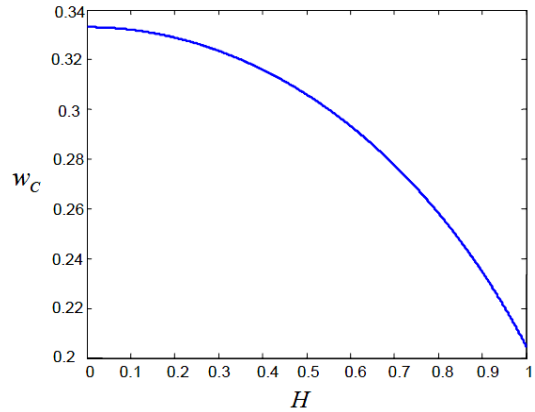


Figure 8. Dependence of QDIA compressive soliton width on H. Other parameters are $u_0=0.3$, $p=9.17$, $\sigma=1$, $d=4$.

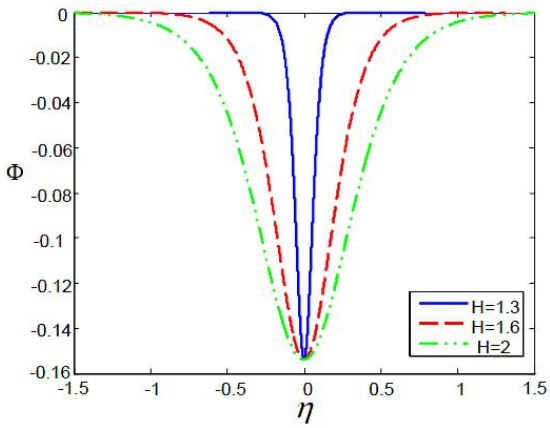


Figure 9. QDIA rarefactive solitons for different values of H, with $u_0=-0.3$, $p=9.17$, $\sigma=1$, $d=4$.

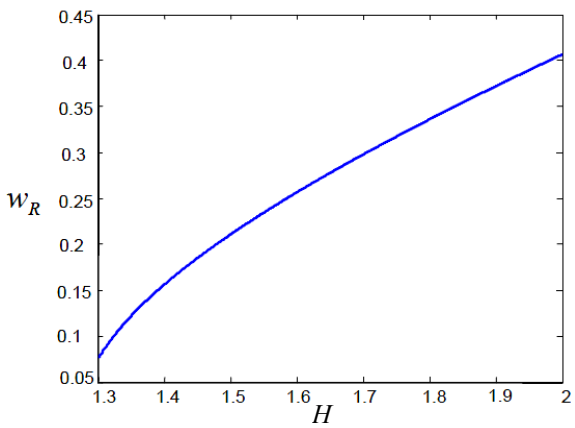


Figure 10. Dependence of QDIA rarefactive soliton width on H. Other parameters are $u_0=-0.3$, $p=9.17$, $\sigma=1$, $d=4$.

5. Results

In this study, the role of dust density and quantum effects on QDIA solitons in an unmagnetized four component quantum plasma have been investigated. KdV equation has been derived and its stationary localized solutions has been obtained. It is shown that the formation of both compressive and rarefactive solitary wave structure is possible in quantum plasma with quantum electrons and positrons, classical cold ions and stationary negative dust grains. There exists a critical value of dust density depending on quantum diffraction parameter, below which compressive soliton and above which a rarefactive soliton formation are possible. Furthermore, it is shown that the amplitude and width of both solitons increase as dust density increases.

Also, the amplitude of both solitons is unaffected by the H but the width of compressive (rarefactive) soliton decreases (increases) as H increase. It is hoped that

current findings can be applicable to highly degenerate dense astrophysical compact objects such as white dwarfs.

Acknowledgments
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