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A novel method describing the space charge limited region in a planar diode

H Ahmady[1], E Mehrshahi[1], and M Ghergherehchi[2,3]
1.Department of Electrical Engineering, Shahid Beheshti University, Tehran, Iran
2.Department of Energy Engineering and Physics, Amirkabir University of Technology, Tehran, Iran
3.College of Electronic and Electrical Engineering, Sungkyunkwan University, Suwon, Korea

E-mail: mitragh@skku.edu

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Abstract
A novel method is presented to describe the physics of space-charge region in a planar diode. The method deals with the issue in the time domain and as a consequence transient time behavior before stabilization can be achieved. Potential distributions and currents obtained using this technique, supposing zero initial velocity for electrons, reveal absolute agreement with other studies results. Moreover, applying the method for non-zero uniform initial velocity for electrons gives the results which are in good agreement with previous works.

Keywords: space-charge region, space-charge limited current, planar diode, initial velocity

1. Introduction

Making different assumptions, the characteristics of space-charge limited region in a planar diode have been studied. The most basic supposition having been made is to consider the electrodes infinitely large and therefore to consider the equipotential surfaces as planes parallel to the electrodes. Based on this presumption and also assuming zero velocity for the electrons emitted, Child [1] and Langmuir [2] obtained the well known three halves law and a formula for potential distribution. Langmuir and Blodgett [3,4] followed the same procedure, however with different geometries. In one of their studies, they calculated space charge between coaxial cylinders [3], and in another study, they calculated it for Concentric Spheres [4].

Extending the classical one-dimensional Child-Langmuir law to two-dimentional law has been done in some studies [5-7]. Two-dimensional space-charge limited current in cylinders with finite length emitter has been studied by Kostov and Barroso [8]. In Luginsland et al. study [9], the analytic and computational extensions to the one-dimensional Child–Langmuir law was reviewed. Koh et al. [10] presented a three-dimentional model of Child-Langmuir law for hot electron emission in planar and cylindrical gap, including finite emission energy effects [10]. Other scholars [11-13] studied quantom extentions of Child-Langmuir law by considering the interactions between electrons.

An experimental investigation of space-charge-limited flow of current in a nanogap was presented in Bhattacharjee et al. [14]. When distance is comparable to wavelength, the current density varies as square root of applied voltage, unlike the classical Child-Languir law [14]. Exact relativistic solutions for the one-dimensional space-charge-limited diode was given in Jori and Trivelpiece [15]. The effects of short pulses and THz frequencies are studied and presented in some other studies [16-18]. In Zhu and Ang [19] study classical Child-Langmuir law was extended to the Coulomb blockade regime.

The space charge current, assuming Maxwellian distribution for initial velocity of electrons was studied by Fry and Langmuir [20,21]. Through a simplified theory, Parsons [22] presented an Exact integration of the space charge equation in a planar diode. The effects of finite initial velocity was examined in Akimov et al. [23] and Ahmady et al. [24]. Extensive and continued suervy has already been carried out on space charge. These efforts which last over a century shows the importance of the topic.

Space charge, is also an important factor in other fields of physics, like beam dynamics of cyclotrons and other particle accelrators (e.g. [25, 26]). Realizing this,

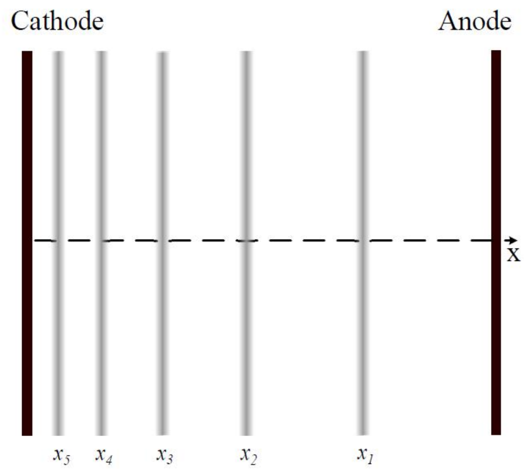

Figure 1. The electron beam modeled by plates of electric charge entering the space.

we have proposed a different approach describing the space-charge limited region. The method is consistant with physics of the problem and is easy to apply. The method can be applied for non-zero initial velocity. Moreover, transient results can be achieved.

2. Formulation

Modeling the problem with entering planes is nearly a usual way, e.g. [19]. In Zhu and Ang [19] study another aspect of the problem, which is finding a threshold voltage for single electron injection is addressed. The situation in the present study is schematically illustrated in figure 1.

The electron beam is modeled by plates of electric charge periodically entering the space between the electrodes. The positions of the charge plates are represented by x_1, x_2, ... The plates are allowed to move only in the x direction and all have the same initial velocity (u_0). The cathode is at potential zero and the anode is at potential V_0. As the charge plates enter the inter-electrode space, the presence of each plate alters the electric field in the space. Assuming infinitely large plates and charge density (σ_{ins}) on them, the difference between electric field on the right and the left of the plates is $\sigma_{ins} / \varepsilon_0$, where ε_0 is the vacuum permittivity. The emission of the electrons is modeled by periodic entrance of plates so that we would be able to observe the transient behavior. The transient behavior of the system is described as follows:

Before emission starts, there is no electron in the space, the potential distribution is linear and the electric field is constant between the electrodes. The entrance of the first group alters the electric field. The motion of the plate is determined according to the force imposed by the electric field. As the next plate enters, the electric field gets modified. The two plates now move in accordance with the modified electric field. The process goes on, plates enter periodically, electric field changes and their position alter correspondingly. The plates enter the space until the system reaches the steady state. In this state, the positions of the available plates in the space are almost fixed and the space charge has been formed.

Consider N to be the number of charge plates in the space. As the first plate enters N value is 1. After the presence of the second plate, N value would be 2. N value increases as the number of entering plates increases, until the system reaches steady state. From this point on, the entrance of plates into the space would not raise N value. Imagine at a time there are N plates in the space. The electric field between $x_n - 1$ and x_n, denoted by E_n, is calculated as :

$$E_n = (-\sigma - \frac{x_1 + \ldots + x_N}{d}\sigma_{ins} + (n-1)\sigma_{ins}) / \varepsilon_0 \ , \tag{1}$$

$n = 1, 2, \ldots, N$,

where σ is the charge density on the electrodes before the entrance of any plate and d is the distance between the electrodes. Using Newton's second law (eqs. 2 and 3), the equation of motion of the nth plate can therefore be described as eq. (4):

$$F_n = m\ddot{x}_n \ , \tag{2}$$

$$F_n = -eE_n \ , \tag{3}$$

$$\ddot{x}_n = \frac{e}{m\varepsilon_0}(-\sigma - \frac{x_1 + \ldots + x_N}{d}\sigma_{ins} + (n-0.5)\sigma_{ins}) \ , \tag{4}$$

$n = 1, 2, \ldots, N$,

where F_n is the applied force and E_n is the applied electric field on the electron. Also, e and m are the charge and mass of electron, respectively.

As the plates enter, their position is calculated using eq. (1). The current can be calculated from their arrival on the anode. The total current density between the arrivals of the nth and (n+1)th plate is composed of two components: J_1, which is due to the arrival of a plate with charge density σ_{ins} on the anode and J_2, the displacement current. These components are calculated as follows:

$$J_1 = \frac{\sigma_{ins}}{t_{arr,n+1} - t_{arr,n}} \ , \tag{5}$$

where $t_{arr,n}$ and $t_{arr,n+1}$ are the arrival times of the nth and (n+1)th plate, respectively.

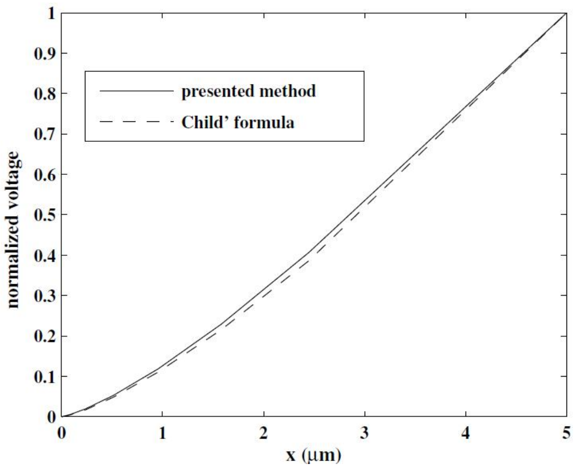

Figure 2. Normalized potential distribution obtained from Child's method vs. presented method $(V_0 = 0.1V, d = 5\mu m, u_0 = 0)$

$$J_2 = \frac{(\frac{\sum x_i}{d}\sigma_{ins})_{t_{arr,n+1}} - (\frac{\sum x_i}{d}\sigma_{ins})_{t_{arr,n}}}{t_{arr,n+1} - t_{arr,n}} , \quad (6)$$

where, in the nominator, the summations hold for available x_i s in the space at times $t_{arr,n+1}$ and $t_{arr,n}$ At steady state, the x_n s would remain unchanged, J_2 approaches zero and the current density would actually be J_1. The potential at the steady state can be obtained integrating the electric field.

In addition to the above mentioned way, the current can be obtained from the below formula:

$$J = \rho v. \quad (7)$$

We expect that in steady state, ρv would be the same for all points and be the same as J_1. The results presented in comprisonal results section (4.) live up to this expectation.

3. Procedures

The procedure of the method is as follows. The first plane was emitted from the cathode and entered the space. The motion of this plane is described by eq. (4) where n is equal to *1*. Using Laplace transform, the position and velocity of this plane at T seconds was obtained when the second plane entered. As the second plane entered, the electric fields and consequently the forces applied to planes changed. Making use of eq. (4) and Laplace transform, the positions and velocities of these two planes at the time of the arrival of the third plane were determined. The procedure went on this way. Planes entered into the space one after another and their positions and velocities were calculated. As the planes collided the anode, the arrival current was calculated using eq. (5). Displacement current was calculated using eq. (6). Finally, the total current was the sum of these two current.

4. Comparisonal results

As an illustrative example, the technique presented in the previous section was applied to a system with $V_0 = 0.1V$, $d = 5\mu m$, $T = 21ns$, $\sigma_{ins} = 0.12\sigma$ and zero initial velocity for emitted electrons. The normalized potential distribution and current density, are shown in figures 2 and 3, respectively.

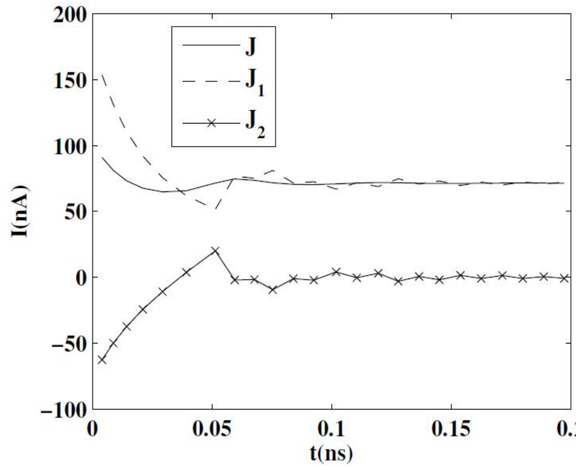

Figure 3. Current density J and components J_1 and $J_2 (V_0 = 0.1V, d = 5\mu m, u_0 = 0)$.

To be able to make a comparison, the potential distribution obtained from Child's formula is also drawn in figure 2. Clearly, the two plots are almost congruent. Child-Langmuir formulas for space-charge limited current and potential are given by Child [1] and Langmuir [2]:

$$I_{CL} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m}}V_0^{3/2} , \quad (8)$$

$$V_{CL}(x) = (\frac{x}{d})^{4/3} V_0 . \quad (9)$$

For the above system, I_{CL} is 73 nA. As can be seen in figure 3, the current at the steady state reached I_{CL}. To consider the effect of non-zero initial velocity, the technique was applied to a system with the above parameters, but non-zero initial velocity was equal to 0.1mV for emitted electrons. Using either of the two mentioned ways, the current at steady state would be 81.3nA.

In Akimov et al. [23], the space-charge limited current with non-zero initial velocity is shown to be:

$$I_{SCL} = I_{CL}[(\frac{mu_0^2}{2eV_0})^{1/2} + (1 + \frac{mu_0^2}{2eV_0})^{1/2}]^3 . \quad (10)$$

Making use of the above equation, space-charge limited current would be 81.1 nA, showing good agreement with the result of our method.

5. Conclusion

In this paper, the electron beam was modeled by plates of electric charges. Applying the method gave reasonable results in agreement with previous works. The advantage of the method lies in the fact that it is simple and provides transient time results. The method can also be applied for non-zero initial velocity emission.

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