The effect of material nonlinearity on the band gap for TE and TM modes in square and triangular lattices

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Abstract

In this article, by using the method of finite difference time domain (FDTD) and PML boundary conditions, we have studied the photonic band gaps for both TE and TM modes in square and triangular lattices consisting of air holes in the dielectric medium; the results have been compared too. In addition, the effect of the nonlinearity of the photonic crystal background on the photonic band gaps and comparison with the results of the linear case (holes in a background medium with the linear dielectric constant) have been presented. Comparison of the transmission spectra in the linear and nonlinear cases shows a red shift in the minimum transmission for both triangular and square lattices

Keywords: photonic crystals, nonlinear materials, band gaps, TE and TM modes

1. Introduction

There is a lot of ongoing research in the area of photonic band gap structures or photonic crystals (PC) [1]. These PCs are periodic structures that could manipulate beams of light in the same way that semiconductors control electric currents. A semiconductor cannot support electrons of energies falling within the electronic band gap. Similarly, a photonic crystal cannot support photons lying in the photonic band gap. By preventing or allowing light to propagate through a crystal, light processing can be done. By introducing defects or nonlinearity, various circuits may be designed. The study includes, on the one hand, the research of materials and structures that are best suited for this purpose [2]; on the other hand, the development (or improvement) of numerical tools is discussed [3].

A photonic crystal can be made either by arranging a lattice of air holes on a transparent background dielectric or by forming a lattice of a high refractive index material embedded in a transparent medium with a lower refractive index [4, 5]. The lattice size may be roughly estimated to be the wavelength of light in the background medium. The existence of the photonic band gap in the photonic crystals implies the possibility of

spontaneous emission control and the potential applicability to optoelectronic devices such as zero-threshold laser, high-efficient light emitting diodes, and low-loss waveguides [6].

A two-dimensional crystal is a periodic array of rods or holes in a background medium. The lattice constant in photonic crystals demonstrates the minimum length along which a rod-hole configuration repeats itself. The most important effect of the periodicity in the photonic crystals is the existence of band gaps in frequencies for which the light propagation is forbidden [1, 7, 8]. Maksymov, and Marsal, have simulated and studied the nonlinear photonic crystals in 2-D with holes in a background medium for TE and TM modes [8]. They showed that band gaps, in the nonlinear medium, in comparison with the linear medium, are shifted to a higher wavelength (see also the results in ref. [9]). Han-Youl Ryu *et al*. have also studied the effects of size non-uniformities in two-dimensional photonic crystals by using the plane- wave expansion. They have investigated the square, triangular, and graphite arrays of dielectric rod or air holes [10]. We entered the effect of Kerr nonlinearity in the finite-difference time domain (FDTD) code [8]; then this effect was studied on the photonic band gaps in the transmission spectra of various
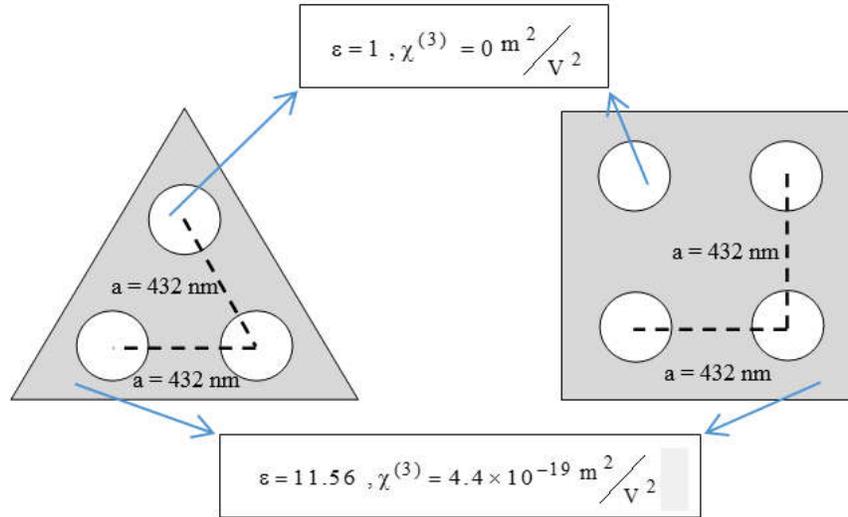
Figure 1. Unit cells of square and triangular lattices.

wavelengths. The remaining parts of the paper are organized as follows: section 2 is devoted to formalism. In section 3, the square and triangular lattices of our interest are defined and the transmissions in both arrangements are calculated for both TE and TM modes. The results obtained for the linear and nonlinear materials are compared. Finally, in section 4, the final remarks and conclusions will be presented.

2. Maxwell's Equations and the Equivalent Set of Finite Difference Equations

The computational method we used to simulate photonic crystal structures is the time domain method. The simulation is based on the well-known FDTD technique. The FDTD method is a rigorous solution to Maxwell's equations and does not have any approximations or theoretical restrictions. This method is widely used as a propagation solution technique in the integrated optics. FDTD is a direct solution of Maxwell's curl equations and therefore, includes many more effects than a solution of the monochromatic wave equation.

The Maxwell's equations in an isotropic medium [8] are as follows:

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \qquad (1a)$$

$$\vec{D}(\vec{r},t) = \varepsilon \vec{E}(\vec{r},t), \vec{B}(\vec{r},t) = \mu \vec{H}(\vec{r},t) , \qquad (1b)$$

These equations in the rectangular coordinate system are equivalent to the following system of scalar equations:

$$\frac{\partial B_z}{\partial t} = (\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}), \text{etc}. \qquad (2a)$$

$$\frac{\partial D_x}{\partial t} = (\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) , \qquad (2b)$$

$$\frac{\partial D_y}{\partial t} = (\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) , \qquad (2c)$$

$$\frac{\partial D_z}{\partial t} = (\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) , \qquad (2d)$$

Maxwell's equations describe a situation in which the

temporal change in the E-field is dependent upon the spatial variation of the H field, and vice versa. The FDTD method solves Maxwell's equations by first discrediting the equations via central differences in time and space and then numerically solving these equations in the software [9].

We denote a grid point of the space by :

$$(x, y, z, t) = (i\Delta x, j\Delta y, k\Delta z, n\Delta t) \equiv (i, j, k, n) , \qquad (3a)$$

$$E_z(x, y, z, t) = E_z(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = E_z(i, j, k, n)$$
$$\equiv E_z^n(i, j, k) , \qquad (3b)$$

From this, a convenient set of finite difference equations for E-field component [Eq. 1a] will be obtained as:

$$\frac{B_x^{n+1/2}(i, j+1/2, k+1/2) - B_x^{n-1/2}(i, j+1/2, k+1/2)}{\Delta t}$$
$$= (\frac{E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k)}{\Delta z} \qquad (4)$$
$$- \frac{E_z^n(i, j+1, k+1/2) - E_z^n(i, j, k+1/2)}{\Delta y}) ,$$

Similarly, a convenient set of finite difference equations for the B-field component [Eq. 1a] can be obtained as

$$\frac{D_x^n(i+1/2, j, k) - D_x^{n-1}(i+1/2, j, k)}{\Delta t})$$
$$= (\frac{H_z^{n-1/2}(i+1/2, j+1/2, k) - H_z^{n-1/2}(i+1/2, j-1/2, k)}{\Delta y} \qquad (5)$$
$$- \frac{H_y^{n-1/2}(i+1/2, j, k+1/2) - H_y^{n-1/2}(i+1/2, j, k-1/2)}{\Delta z}) ,$$

The assumed boundary condition Perfectly Matched layers (PML) suppresses the reflection of the outgoing waves, creating an infinite space for wave scattering.

3. Photonic crystal with square and triangular lattice

The square and triangular lattices consist of air holes in the linear dielectric background with the dielectric constant $\varepsilon=11.56$, and in nonlinear Kerr dielectric background with the characteristics $\varepsilon=11.56$ and $\chi^{(3)} = 4.4 \times 10^{-19} \text{ m}^2/\text{V}^2$ (figure. 1). The radius of the
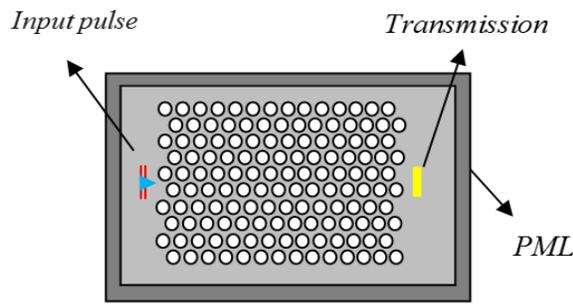
Figure 2. (Color online) Photonic crystal of triangular lattices.
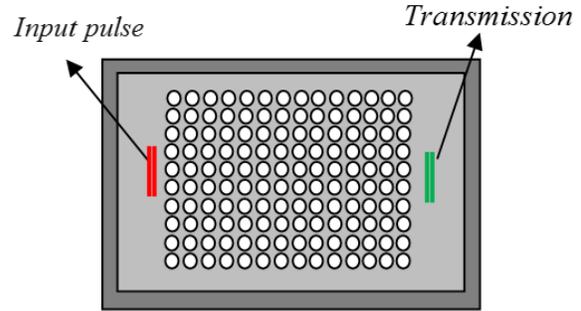

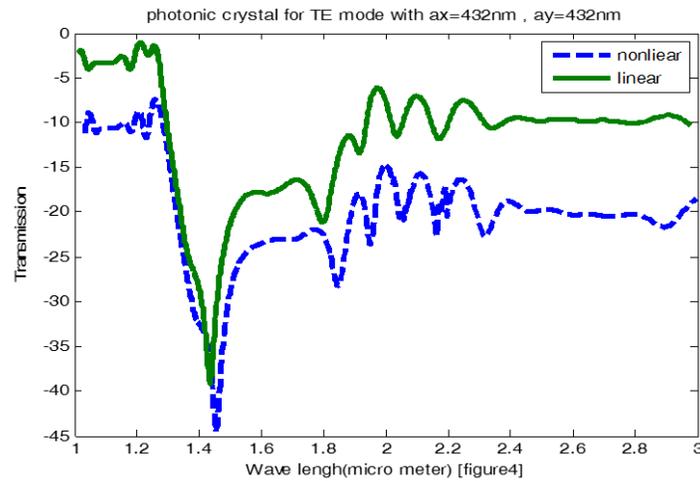**Figure 3**. (Color online) Photonic crystal of square lattices.

Figure 4. (Color online) Light transmission of linear (dark lines) and nonlinear (dashed lines) photonic crystals for the TE mode with the square lattice.
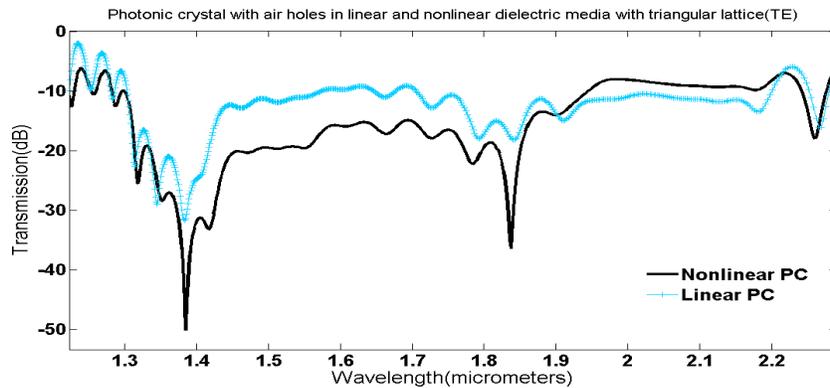

Figure 5. (Color online) Light transmission of linear (dashed lines) and nonlinear (dark lines) photonic crystals for the TE mode with the triangular lattice.

holes is $r = 0.2a$, where a is a lattice constant. For the square lattice, $a = a_x = a_y = 432$ nm, and triangular lattice, $a = 432$ nm. Also, we have used the Gaussian modulated pulse with the central wave length, $\bar{\lambda} = 1550$ nm, in our FDTD code [9].

The modulated Gaussian pulse has been separately applied to square and triangular lattices in figures 2 and 3.

The results for the TE mode in the square lattices are illustrated in figure 4 and the same is done for triangular lattices in figure 5. According to the spectrum of light transmission in linear and nonlinear crystals, a shift in

the wave length appears. Likewise, a decrease in light transmission occurs due to the presence of the nonlinear material in the triangular lattice. One more result was a gap in a range from 1.8 to 1.9 microns.

Figures 6 and 7 demonstrate the light transmission of linear and nonlinear photonic crystals for the TM mode in square (figure 6) and triangular crystals (figure 7). By comparing the spectrum of light transmission in the linear and nonlinear crystals, a shift in wave length for triangular and square lattice was noticed. Also, for all the considered lattices, we found a decrease in light transmission by the presence of nonlinear materials for the both TE and TM modes, but this decrease in
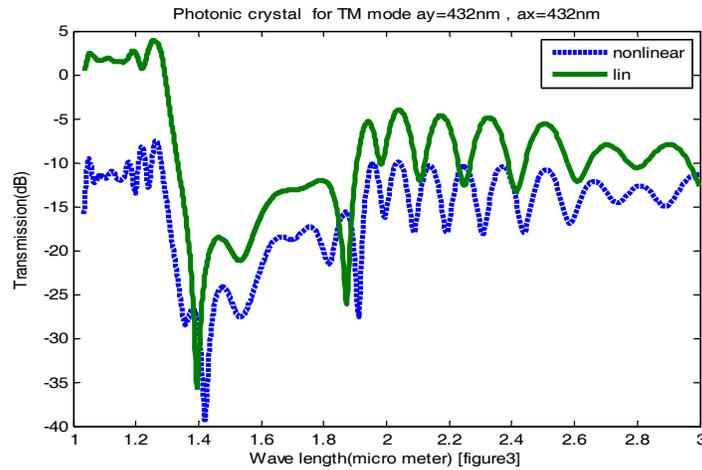
Figure 6. (Color online) Light transmission of the linear (dark lines) and nonlinear (dashed lines) photonic crystals for the TM mode in a square lattice.
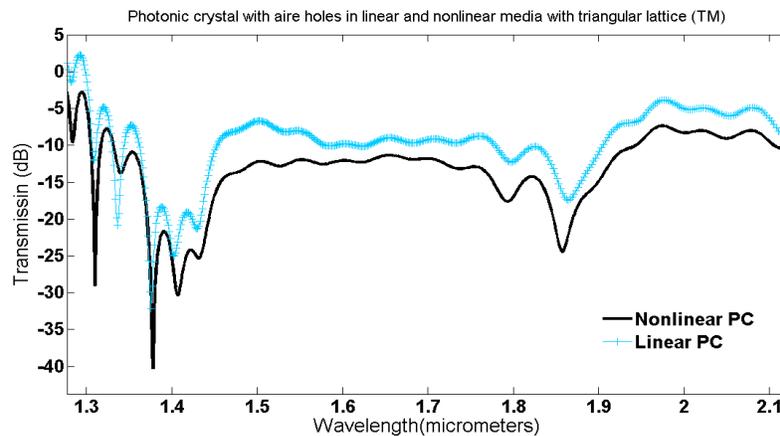

Figure 7. (Color online) Light transmission of linear (dashed lines) and nonlinear (dark lines) photonic crystals for the TM mode with the triangular lattice.

square lattice as well as the TM mode was smaller than that for the others. For the TE mode, in the triangular lattice, there was a gap which ranged from 1.8 to 1.9 microns.

Concluding remarks
We have used the FDTD algorithm and appropriate boundary conditions to solve the Maxwell's curl equations in Cartesian coordinates and obtained the light transmission of the linear and nonlinear photonic crystals for the TM and TE modes in the square and triangular lattices. By comparison, we found the appearance of a shift in wave length in the spectrum of light transmission in the linear and nonlinear crystals as a result of the nonlinearity properties. One more result was a gap limit with a range from 1.8 to 1.9 microns.

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