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# Time dependent gravitational constant in Chern Simons modified gravity

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**Abstract**

Two dark energy models $\Lambda \sim \left(\dot{a}/a\right)^2$ and $\Lambda \sim \ddot{a}/a$ are studied by taking into account the gravitational constant G is a time-dependent parameter in the framework of Chern-Simons modified gravity. It is found that the gravitational constant shown the increasing behavior proportional to those of the time parameter for each model. These models are compared with observational results by regulating the values of the parameters. Our investigations indicated that the model $\Lambda \sim \left(\dot{a}/a\right)^2$ is generally attractive in nature while the other model $\Lambda \sim \ddot{a}/a$ coincides to the repulsive situation and consequently match with the current scenario of the accelerating universe. We calculated the variation of $G$(t) which showed that it changes rapidly when the value of $\omega$ is taken between the limit $-1.33 < \omega < -0.79$. It is viewed that due to the composite influence of time-variable $\Lambda$ and $G$(t), the universe expanded with acceleration. Further, it is estimated that the range for variation of $G$(t) with proper tuning of parameters $\alpha$ and $\beta$ is given *as* $-\left(1.89 \pm 0.10\right)\times 10^{-11}\,\mathrm{yr}^{-1} < \dot{G}/G < 0$, which match with Ia type supernova.

**Keywords:** dynamical CS modified Gravity, dark energy, gravitational constant

## 1. Introduction

Current cosmological observations advocated that the universe is currently going through an accelerated phase of expansion. the observations such as cosmic microwave background radiation [1] and sloan digital sky survey [2] predicted that the universe had a negative-accelerated phase of expansion in ancient times to facilitate structure arrangement of it. this phenomenon is commonly named dark energy (de) posses large negative pressure supposed to comprise about 68% contents of the universe, 27% dark matter and only 5% byronic matter [3]. the nature of de is a big mystery for the cosmologists and astrophysicists. in this scenario, many efforts have been done to resolve this long-standing issue but an acceptable answer is still a dream. different modifications in general relativity have been suggested such as $f(R)$ gravity, $f(T)$ gravity, Gauss-Bonnet theory, Lovelock gravity, scalar-tensor theories and Chern Simons (CS) modified gravity theory, etc.

CS modified gravity initially developed in 3-dimensions by Jackie and Pi [4] elongated it into 4-dims by adding an external scalar field. A number of holographic DE models have been discussed in this theory. Pasqua, *et al*. [5] analyzed in detail the holographic DE model with Granda-Oliveros cut-off, modified holographic Ricci DE model and a model with higher-order derivatives of the Hubble parameter. Myung [6] revisited Ricci DE in CS modified gravity. He made an assumption that the cosmological evolution is nothing but the Ricci DE with a minimally coupled scalar without potential means that the role of CS term is suppressed. Li. *et al*. [7] put observational constraints on the interaction and spatial curvature in the holographic DE model. They considered three kinds of phenomenological interactions between holographic DE and matter, i.e., the reciprocation term $Q$ is proportional to the energy densities of DE ($\rho_m$), matter ($\rho_\Lambda$), and matter plus DE ($\rho_m + \rho_\Lambda$).

Huang and Gong [8] used the type Ia supernova data to investigate the model of holographic DE. For $d = 1$, they got best fit result $\Box^0_m = 0.25$, the equation of state for holographic DE $\omega^0{}_\Lambda$= -0.91 and the evolution between the decelerating expansion and accelerating expansion occurred when the cosmological red-shift was $z_T$ = 0.72. Elizalde with his research fellows [9] planed the structure of DE models with an efficient phantom phase but without exotic matter. They also generalized holographic model that was produced in the presence of an infrared cut-off.

Jawad and Sohail [10] discussed the DE phenomenon by considering the modified QCD ghost DE in the framework of dynamical CS modified gravity. They found an analytical solution of scale factor and investigated different cosmological parameters in this set-up. They also showed that the deceleration parameter identifies different states of the universe under certain conditions of constant parameters. Ali and Amir [11-13] studied some holographic DE models in the framework of CS modified gravity by considering the FRW universe. They examined the equation of state parameter using Granda and Oliveros infrared cut-off proposal which expressed the accelerated expansion of the universe in the context of CS modified gravity theory.

Porfirio *et al*. [14] established the correspondence for G□del-type solutions within the 4-dimensional CS modified gravity with the non-dynamical CS coefficient for various forms of matter such as dust, fluid, scalar field, an electromagnetic field, and related causality problems. Konno *et al*. [15] discussed rotating black hole solutions in the framework of CS modified gravity theory by taking a description of agitation around the Schwarzschild solutions. Guarrera and Hariton [16] designed a preserved, symmetric energy-momentum pseudo tensor using CS modified gravity which showed that it is Lorentz invariant. Nandi *et al*. [17] analyzed the impact of CS modified gravity on the quantum phase shift of de Broglie waves in neutron interferometry. Chen and Jing [18] investigated the geodesic precession and the strong gravitational lensing in the slowly rotating black hole in the dynamical CS-modified gravity theory.
This article is organized in the following order: In section 2, we wrote the brief introduction and formalism of Chern-Simons modified gravity. Dark energy models and basic field equation of CS modified gravity theory are constructed in section 3. We proceed the study to two different DE models and investigated them in secion 4 and 5. Last section is devoted for summery and conclusions.

## 2. Formulism of Chern-Simons Modified Gravity

An impressive principle of modification of GR is CS modified gravity theory which developed on the leading-order gravitational parity violation. It is inspired by peculiarity cancellation in particle physics as well as string theory. The Einstein Hilbert action is modified by adding CS and scalar field terms.

$$S = S_{EH} + S_{CS} + S_\Theta + S_{mat} \tag{1}$$

Where Einstein Hilbert term is denoted as

$$S_{EH} = \kappa\int_v d^4x\sqrt{g}R \tag{2}$$

CS term is represented as

$$S_{CS} = +\alpha\frac{1}{4}\int_v d^4x\sqrt{-g}\Theta\,{}^*RR \tag{3}$$

term scalar field expressed

$$S_\Theta = \beta\frac{1}{2}\int_v d^4x\sqrt{-g}\left[g^{ab}\left(\nabla_a\Theta\right)\left(\nabla_b\Theta\right)+2V(\Theta)\right] \tag{4}$$

an additional undefined matter contributions is given as

$$S_{mat} = \int_v d^4x\sqrt{-g}\,£_{mat} \tag{5}$$

Where $£_{mat}$ represents some matter Lagrangian density, $\kappa = \frac{1}{16\pi\,G}$, g is determinant of metric, $\nabla_a$ covariant derivative, R is a Ricci scalar and integrals represent the volume executed anywhere on the manifold. Pontryagin density ${}^*RR$ is mathematically given as

$${}^*RR = R\tilde{R} = {}^*R_b^{acd}R_{acd}^b \tag{6}$$

Dual Riemannian tensor defined as

$${}^*R_b^{acd} = \frac{1}{2}\epsilon^{cdef}R_{bef}^a \tag{7}$$

where $\epsilon^{cdef}$ four dimensional Levi-Civita tensor. Formally, ${}^*RR \propto R\Lambda\, R$, however, the curvature tensor is supposed to be Riemannian tensor.
Now the variation of action w.r.t metric $g_{ab}$ and scalar field Θ, we obtained set of field equations of CS modified gravity in the following form

$$G_{ab} + \alpha C_{ab} = -\frac{1}{2\kappa}\left(T_{ab}^m + T_{ab}^\theta\right), \tag{8}$$

$$g^{ab}\nabla_a\nabla_b\Theta = -\frac{\kappa\alpha}{4}{}^*RR \tag{9}$$

where $G_{ab}$ is Einstein tensor, α coupling constant, $C_{ab}$ is cotton tensor defined as

$$C^{ab} = -\frac{1}{2\sqrt{g}}\left[v_\sigma\epsilon^{\sigma\mu\zeta\eta}\nabla_\zeta R_\eta^\nu + \frac{1}{2}v_{\sigma\tau}\epsilon^{\sigma\nu\zeta\eta}R_{\zeta\eta}^{\tau\mu}\right] \tag{10}$$

where $v_\sigma = \nabla_\sigma\,\Theta$ and $v_{\sigma\tau} = \nabla_\sigma\nabla_\tau\;\;\Theta$. The energy momentum tensor $T_{ab}$ comprises of matter part and external field part defined as

$$T_{ab}^m = \left(\rho + p\right)U_aU_b - pg_{ab}\,, \tag{11}$$

$$T_{ab}^\Theta = \eta\left(\partial_a\Theta\right)\left(\partial_b\Theta\right) - \frac{\eta}{2}g_{ab}\left(\partial^\alpha\Theta\right)\left(\partial_\alpha\Theta\right) \tag{12}$$

Here p, ρ and U are pressure, energy density and the four-vector velocity in co-moving coordinates of the spacetime.

The Chern-Simons gravitational modification has been mostly studied in the non-dynamical context. In this context, the scalar field is non-dynamical thus it is supposed to be prior prescriptive function of spacetime. This type of investigations is mostly introduced in the evaluation of approximate solutions, sometimes exact solution, cosmological study, astrophysical tests and matter interactions.

In non-dynamical CS modified gravity theoretically problematic association between Schwarzschild black hole perturbation theory, the occurrence of static and axisymmetric solutions and uniqueness of solutions theory, has been demonstrated. But there are numerous issues in non-dynamical theory:- (i) During the rotation of black hole singularities of curvature to be seen on the rotational axis, (ii) oscillation modes of massive group of black hole is hidden, (iii) Commonly ghost arises. Therefore, the dynamical CS modified gravity including kinetic term for scalar field is prescribed by several authors to overcome above mentioned issues and to preserve the self-stability of the theory. Problems (i) and (ii) mentioned above do not arise in the dynamic theory and last one does not occur in a particular conditions. Consequently, the dynamical CS modified gravity has captured more interest in recent age.

## 3. Dark Energy Models in CS Modified Gravity

The appearance of objects at cosmological distances is affected by the curvature of spacetime through which light travels on its way to earth. Einstein's theory of relativity entirely describes the geometrical properties of the universe. Metric is a fundamental quantity in GR which characterizes the geometry of spacetime. The curvature of space may deviate with time in the homogeneous and isotropic universe, at a given time its value remains uniform everywhere since Big Bang. In the mid of 1930, Robertson with his co-workers independently proved that the FRW metric is the most general metric for describing the expanding homogeneous isotropic universe. A homogeneous and isotropic universe expressed by the FRW metric is given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{1}{1-\kappa r^2}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \tag{13}$$

The field equations of CS modified gravity in the presence of cosmological constant Λ are given by

$$G_{\mu\nu} + lC_{\mu\nu} = -8\pi G\left[T^m_{\mu\nu} - \frac{\Lambda}{8\pi G}g_{\mu\nu}\right] + T^{\Theta}_{\mu\nu} \tag{14}$$

The 00-components of Eq. (14) are given as

$$3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3k}{a^2} - \Lambda - \frac{1}{2}\dot{\Theta}^2 = 8\pi G\rho. \tag{15}$$

Using Eq. (9), we explored the value of external field Θ in the dynamical CS gravity theory. It is mentioned here that all the components of Cotton tensor are turned to be zero for FRW metric, it is also noted that the Pontryagin term becomes zero identically so Eq. (9) reduces to

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Theta = g^{\mu\nu}\left[\partial_\mu\partial_\nu\Theta - \Gamma^{\beta}_{\mu\nu}\partial_\beta\Theta\right] = 0 \tag{16}$$

In dynamical case Θ is a function of spacetime coordinates. For the sake of simplicity, we consider Θ is a function of time parameter and hence evaluated as

$$\dot{\Theta} = ca^{-3} \tag{17}$$

Substituting this value in Eq. (15) and taking into account that universe is flat, one arrived at

$$3\left(\frac{\dot{a}}{a}\right)^2 - \Lambda - \frac{1}{2}ca^{-6} = 8\pi G\rho. \tag{18}$$

The 11-component of Eq. (14) is given by,

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi Gp + \frac{\Lambda}{2} \tag{19}$$

Solving Eq. (18) and Eq. (19) simultaneously it turned out

$$3\left(\frac{\ddot{a}}{a}\right) = 4\pi G\left(\rho + p\right) + \Lambda \tag{20}$$

Since the Hubble parameter $\frac{\dot{a}}{a} = \mathrm{H}$. In term of Hubble parameter the Eq. (18) and (20) we obtained

$$4\pi G\rho = \frac{1}{2}\left[3H^2 - \Lambda - \frac{1}{2}ca^{-6}\right], \tag{21}$$

$$3\left(H^2 + \dot{H}\right) = -4\pi G(\rho + 3p) + \Lambda \tag{22}$$

## 4. Model with $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$

A number of DE models have been suggested to discuss the universe [19-21]. In this paper, we took a model $\Lambda = \left(\frac{\dot{a}}{a}\right)^2 = 3\alpha\,\mathrm{H}^2$, where α is the free parameter [22] and substituting in Eq. (21) such that

$$4\pi G\rho = \frac{3(1-\alpha)}{2}H^2 - \frac{ca^{-6}}{4} \tag{23}$$

The barotropic equation of state p=ωρ, here ω is the parameter equation of state and depends on time, redshift or scale factor in general. Making the use of barotropic EoS in Eq. (22), we obtained the following result

$$3\left(H^2 + \dot{\mathrm{H}}\right) = 4\pi G\rho\left(1 + 3\omega\right) + \alpha H^2. \tag{24}$$

From Eq. (23), put the value of $4\pi G\rho$ in Eq. (24) we have

$$3\left(H^2 + \dot{\mathrm{H}}\right) = \left[\frac{3(1-\alpha)}{2}H^2 - \frac{ca^{-6}}{4}\right](1 + 3\omega) + \alpha H^2 \tag{25}$$

The simultaneous solution of Eq. (23) and Eq. (25) look like to be

$$\dot{\mathrm{H}} = \frac{3(1-\alpha)(1+\omega)}{2}H^2 - \frac{(1+3\omega)}{12}ca^{-6} \tag{26}$$

For analytic solution it can be written in the following form

$$\frac{\ddot{a}}{a} - \frac{2 - 3(1-\alpha)(1+\omega)}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{(1+3\omega)}{12}ca^{-6} \tag{27}$$

This 2nd order differential equation is executed using the method of reduction of order by substituting $\dot{a}$= y in Eq. (27)

$$y\frac{dy}{da} - \frac{2 - 3(1-\alpha)(1+\omega)}{2}\frac{\mathrm{y}2}{a} = \frac{(1+3\omega)}{12}ca^{-5} \tag{28}$$

Again, it can be reduce into linear differential equation by substituting $y^2$= v,

$$\frac{dv}{da} + \frac{-2 + 3(1-\alpha)(1+\omega)}{a}v = \frac{(1+3\omega)}{6}ca^{-5}, \tag{29}$$

which gave the analytic solution as under

$$v = \frac{(1+3\omega)c}{-36 + 18(1-\alpha)(1+\omega)}\frac{1}{a^4} \tag{30}$$

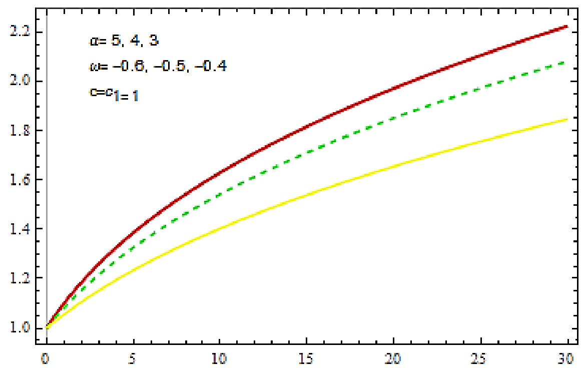


**Figure** 1. The graph of scale factor parameter a(t) versus cosmic time (t).

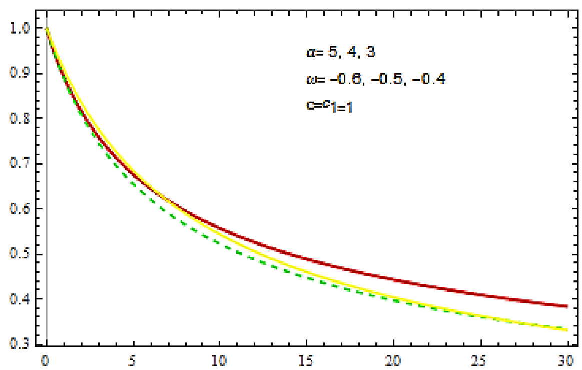


**Figure** 2. Graph of energy density parameter ρ(t) versus cosmic time t.

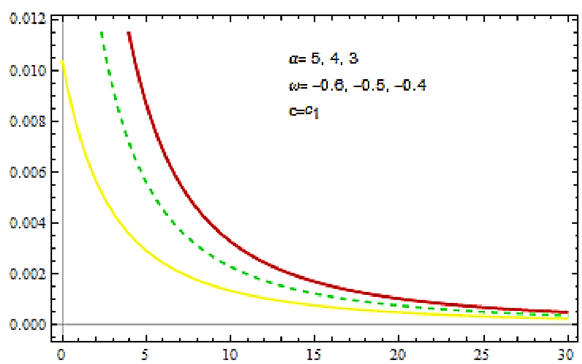


**Figure** 3. Graph of DE model parameter Λ(t) versus cosmic time t.

Using backward substitution and integrating Eq. (30) we arrived at

$$a(t)=\left[3\sqrt{\frac{(1+3\omega)c}{-36+18(1-\alpha)(1+\omega)}}t+c_1\right]^{\frac{1}{3}} \quad (31)$$

where $c_1$ is the constant of integration.

To analysis the behavior of scale factor, authors plotted a graph b/w scale factor "a" and cosmic time "t" for different values of parameters α and ω.

The behavior of scale factor for DE model in CS gravity showed that it increases with cosmic time in each case.

The law of conservation of energy in GR is given by

$$\dot{\rho}+3(\rho+p)\frac{\dot{a}}{a}=0 \quad (32)$$

Substituting p=ωρ in Eq. (32), this first order differential equation is evaluated as

$$\rho=a^{-3(1+\omega)} \quad (33)$$

$\rho$ is the DE density can be explored by using the value of scale factor found in Eq. (31), such that

$$\rho(t)=c_2\left[3\sqrt{\frac{(1+3\omega)c}{-36+18(1-\alpha)(1+\omega)}}t+c_1\right]^{-(1+\omega)} \quad (34)$$

where $c_2$ is constant of integration.

The graphical representation of energy density and cosmic time for different values of α and ω is shown in fig (3, 2). In this graph by fixing $c=c_1$ parameters α and ω are varied as illustrative values of α= 5, 4, 3 and ω= -0.6, -0.5, -0.4 corresponding to red, green and yellow curves respectively.

It is obvious that the of energy density graph for DE model in CS modified gravity showed the decreasing behavior.

Making use of values of scale factor and energy density,

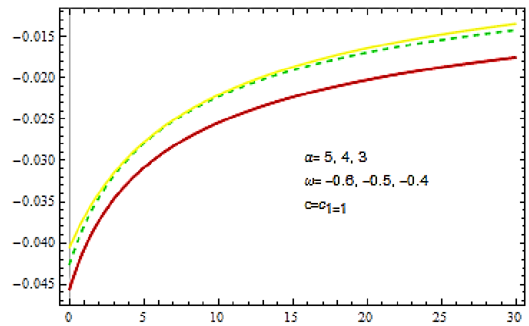


**Figure** 4. Graph of time dependent gravitational constant parameter G(t) versus cosmic time (t).

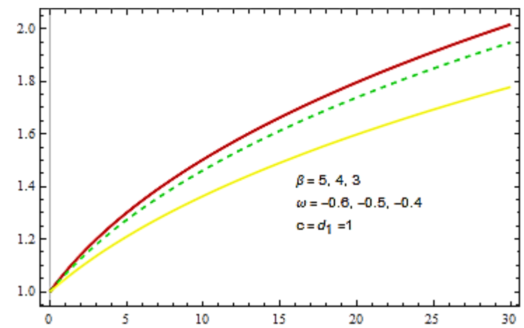


**Figure** 5. Graph of scale factor parameter a(t) versus cosmic time t.

the expression for cosmological parameter adopted as DE model, is turned to be

$$\Lambda(t)=\frac{\alpha(1+3\omega)c}{[-36+18(1-\alpha)(1+\omega)]\left[3\sqrt{\frac{(1+3\omega)c}{-36+18(1-\alpha)(1+\omega)}}t+c_1\right]^2} \tag{35}$$

We plotted a graph for cosmological parameter and cosmic time for various values of α and ω. In this graph by fixing $c=c_1$, parameters α and ω are varied α= 5, 4, 3 and ω= -0.6, -0.5, -0.4 corresponding to red, green and yellow curves respectively.

The graphical behavior of DE model parameter Λ(t) in CS gravity showed that Λ(t) decreases as the cosmic time increases.

Making use of ρ(t) and Λ(t) in Eq. (21), we got the expression for time dependent gravitational constant parameter.

$$G(t)=\frac{3(1-\alpha)}{8\pi c_1}\left[\frac{(1+3\omega)c}{36+18(1\alpha)(1+\omega)}c\right] \times\left(3\sqrt{\frac{(1+3\omega)c}{36+18(1}\alpha}\right)(1+\omega)t+c_1\Bigg)^{(\omega-1)} \tag{36}$$

Fig (4) showed time dependent gravitational constant vs cosmic time graph for different values of α and ω. In this graphical relationship fixing $c=c_1=1$, α= 5, 4,3 and ω= -0.6, -0.5, -0.4, corresponding to red, green and yellow curves respectively.

From the graph of time dependent gravitational constant noted that G(t) started from -0.045 and finally approaches to small value with increasing cosmic time.

## 5. Model with Λ ~$(\ddot{a}/a)$

Let us consider another model $\Lambda=\beta\frac{\ddot{a}}{a}=\beta(H^2+\dot{H})$ [23] where β is a constant, to study the parameters used in pervious section. Substituting this model in Eq. (21) we got

$$4\pi G\rho=\frac{1}{2}\left[(3-\beta)H^2-\beta\dot{H}-\frac{1}{2}ca^{-6}\right] \tag{37}$$

$$(3-\beta)\left(H^2+\dot{H}\right)=\frac{1}{2}[(3-\beta)H^2-\beta\dot{H} -\frac{1}{2}ca^{-6}](1+3\omega) \tag{38}$$

Simultaneous solution of Eq. (37) and (38) give rise to

$$(2-\beta-\beta\omega)\dot{H}=]]-(3-\beta)(1+\omega)H^2 +\frac{(1+3\omega)}{4}ca^{-6}. \tag{39}$$

Now we replace H and $\dot{H}$ with scale factor a and simplify alike terms such that

$$(2-\beta-\beta\omega)\frac{\ddot{a}}{a}+(1+3\omega)\left(\frac{\dot{a}}{a}\right)^2=\frac{(1+3\omega)}{6}ca^{-6}. \tag{40}$$

Using the similar techniques applied in the previous section, one arrived at

$$a(t)=\left[3\sqrt{\frac{(1+3\omega)}{6(-3-2\beta+2\beta\omega+3\omega)}}ct+d_1\right]^{\frac{1}{3}}. \tag{41}$$

where $d_1$ is the constant of integration

The relationship between scale factor and cosmic time for various values of β and ω shown in Fig (5). In this graph by fixing $c=d_1=1$, parameters β and ω are varied as illustrative values of β= 5, 4, 3 and ω= -0.6, -0.5, -0.4 corresponding to red, green and yellow curves respectively.

The behavior of scale factor for DE model in CS gravity showed the increase with cosmic time.

Using the expression of scale factor in Eq. (33), we

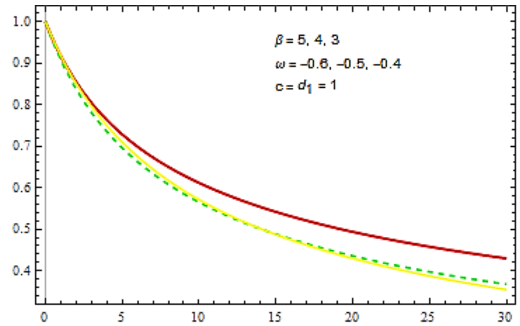


**Figure** 6. Graph of energy density parameter ρ(t) versus cosmic time t.

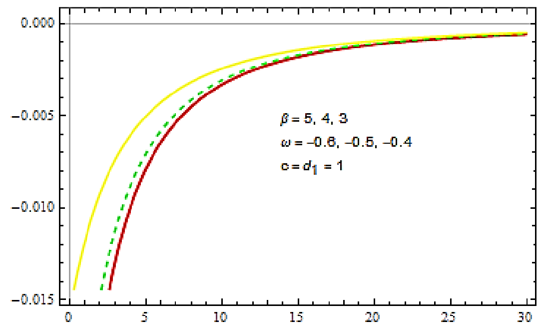


**Figure** 7. Graph of cosmological constant parameter Λ(t) versus time t.

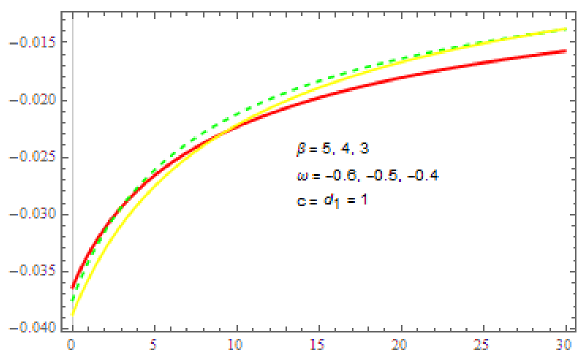


**Figure** 8.The evolution of time dependent gravitational constant parameter G(t) versus cosmic time (t).

obtained energy density ρ (t) given as

$$\rho(t)=\frac{d_2}{\left(3\sqrt{\frac{(1+3\omega)}{6(-3-2\beta+2\beta\omega+3\omega)}}ct+d_1\right)^{1+\omega}} \quad (42)$$

We plotted a graph for energy density and cosmic time using various values of β and ω. Taking $c=d_1=1$ along with β= 5, 4, 3 and ω= -0.6, -0.5, -0.4 corresponding to red, green and yellow curves respectively.

The graphical behavior of energy density for this model in CS gravity showed that ρ(t) starts from high positive value and at late time approaches to enough small.

Using the value of energy density, we obtained the general solution of cosmological parameter.

$$\Lambda(t)=\frac{(1+3\omega)c\beta}{3[-3-2\beta+2\beta\omega+3\omega]\left[\left[3\sqrt{\frac{(1+3\omega)}{6(-3-2\beta+2\beta\omega+3\omega)}}ct+d_1\right]^2\right.} \quad (43)$$

Fig (7) represents graphical relation of cosmological constant parameter and cosmic time for different values of β and ω. Supposed value of β, ω and c are same in each case.

The graphical behavior of cosmological constant parameter for this model in CS gravity shows that Λ(t) decreases as the cosmic time increases.

Making the use of ρ(t) and Λ(t) in Eq. (21) we got the general solution of gravitational constant parameter.

$$G(t)=\frac{1}{8\pi d_2}\left[\frac{(1+3\omega)(3+2\beta)c}{6(-3-2\beta+2\beta\omega+3\omega)}-c\right]\times\left(3\sqrt{\frac{(1+3\omega)}{6(-3-2\beta+2\beta\omega+3\omega}}ct+d_1\right)^{\omega-1}. \quad (44)$$

Where $d_2$ is another constant of integration.

The time dependent gravitational constant vs cosmic time graph for different values of β and ω shown in fig (8). $c=d_1$ keeping constant, the parameters β and ω are varied as illustrative values of β= 5, 4, 3 and ω= -0.6, -

0.5, -0.4 corresponding to red, green and yellow curves respectively.

It had been found that graph of time dependent gravitational constant G(t) increasing behavior with the passage of time.

## 6. Comparison with other Models

From Eq. (36) we have

$$\frac{\dot{G}}{G}=\frac{3\sqrt{P}\left(\omega-1\right)}{3\sqrt{P}t+b} \tag{45}$$

As $\frac{\dot{a}}{a}$ =H so the above expression becomes

$$\frac{\dot{G}}{G}=3\left(\omega-1\right)H \tag{46}$$

Stefaneic [24] discussed the expression for $\frac{\dot{G}}{G}$

$$\frac{\dot{G}}{G}=3\left(1+\eta\right)\Omega_{\Lambda}^{0}H_{0} \tag{47}$$

By the comparison (46) with (47) and using $\omega^0{}_\Lambda= \frac{2}{3}$ from recent observational data we obtain

$$\eta=\frac{3\omega-5}{2} \tag{48}$$

Eq. (48) inter-relates with parameter $\eta$ of the phantom energy model. Stefancic [24] studied the dependence of G(t) on time is more responsive to the value of the phantom energy model parameter $\eta$. In the ancient universe G(t) changed slowly but the variation of G(t) is amazing in the present age meanwhile at large time G(t) approaching zero. It changes rapidly for more negative values of parameter $\eta$. The increasing negative values of $\eta$ proved the suitable ground for testing different models. It would provide the strictest conditions on the growth of cosmological constant, energy density and scale factor depicted by the parameter $\eta$. Stefancic [24] mentioned that the quantity G(t) changes faster with more negative values of $\eta$. In the present work the expression for G(t) showed that it changes rapidly when we use the value of ω between the limit $-1.33 < \omega < -0.79$ .

## 7. Numerical Results

The variability of gravitational constant with time has been proved by a large number of astronomical observations. All these observations agreed with Dyson opinion as he mentioned that variation of gravitational constant G as the order of Hubble parameter H. As $H \propto t^{-1}$, therefore, G decreases as $t^{-1}$. Zhang and Wu [25] proved that current value of $H_0 = 6.64\times10^{-11}\text{yr}^{-1}$ based on the experimental data from WMAP. Cetto *et al*. [26] discussed that astronomical observations according to Brans-Dicke theory in which $G \propto t^{-1}$ as given below.

$$G\left(t\right)=c_1 H\left(t\right)=c_2[H(t)]^{\frac{3}{2}}[\rho(t)]^{-\frac{1}{2}} \tag{49}$$

Guenther *et al*. [27] obtained the range of $\frac{\dot{G}}{G}$ by using the Helioseismological data and observed best range for the variation of G and given as: $1.60\times10^{-11}\,\text{yr}^{-1} < \frac{\dot{G}}{G} < 0$. Damour *et al*. [28] calculated the variation range for $\frac{\dot{G}}{G}$ with the help of data acquired from Binary Pulsar given as: $\left(1.89\pm0.10\right)\times10^{-11}\,yr^{-1} < \frac{\dot{G}}{G} < 0.$ Gaztanaga *et al*. [29] estimated the optimum range for the G variation using the data collected from Ia supernova given as: $-10^{-11}\,yr^{-1} < \frac{\dot{G}}{G} < 0$ Benvenuto [30] estimated another supreme range for the variation of G through astro-seismological data given by: $-2.5\times10^{-11}\,yr^{-1} < \frac{\dot{G}}{G} < +4.5\times10^{-10}\,yr^{-1}$. Biesiada and Malec [31] used the white dwarf star data and determined the best limit for variation of G given as: $\frac{\dot{G}}{G} \le +4.1\times10^{-11}\,yr^{-1}$. Copi *et al*. [32] recently calculated $\frac{\dot{G}}{G}$ using Big Bang nuclei-synthesis as: $-4.1\times10^{-13}\,\text{yr}^{-1} < \frac{\dot{G}}{G} < +3.5\times10^{-13}\,\text{yr}^{-1}$. In the present work we estimate the range for variation of G with the proper tuning of α and β given as $(1.89\pm0.10)\times10^{-11}\,yr^{-1} < \frac{\dot{G}}{G} < 0$ which match with [29].

## 8. Summary and Discussion

In this paper, authors drag out two distinct kinematical DE models in the CS modified gravity theory in the presence of time-dependent gravitational constant G. We evaluated FRW metric in the context of CS modified gravity for scale factor a(t), energy density ρ(t), cosmological constant Λ(t) and gravitational constant G(t). We compared different limits of variations of G acquired from theoretical and observational data with the expressions for $\frac{\dot{G}}{G}$ for both models. We showed that the parameters of both models used in the present work are adjusted in most cases to equate with ranges of $\frac{\dot{G}}{G}$ collected from different sources. All values of $\frac{\dot{G}}{G}$ discussed showed that gravitational constant is inversely proportional to time $(G\sim t^{-1})$. In GR, Belinchon [33] got similar results i.e $G\sim t$ during the dimensional study using Dirac's large number of the hypothesis (LNH) which is clearly opposite to Dirac's conclusion. But in the present study G also varies inversely with t. Conclusively, it is found that expressions for scale factor a(t) and cosmological constant Λ(t) for both models supposed in the present work preserve the same state irrespective of variability of gravitational constant G. we estimate the range for variation of G with the proper tuning of α and β given as $-(1.89 \pm 0.10)\times 10^{-11}\,\mathrm{yr}^{1} < \frac{\dot{G}}{G} < 0$ which match with [29].

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