Iranian Journal of Physics Research, Vol. 21, No. 3, 2021

Research note

Theoretical calculation of angular distribution of the 16O(d,p0)17O reaction at low energies

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(Received 10 March 2020 ; in final form 17 December 2020)

Abstract
Nuclear Reaction Analysis (NRA) is employed to perform depth profiling of the light elements (carbon, nitrogen, oxygen …) in the heavy element substrates. To conduct the analysis, the availability and reliability of the respective differential cross-sections are among crucial factors to achieve the reliable results. In this work, we made a theoretical attempt to calculate the angular distribution of the transfer reaction (stripping reaction) ^{16}O $(d,p_0)^{17}O$ reaction at 1.3 and 1.6 MeV energies in the lab frame, which are usable in NRA. Moreover, the optical potential parameters, which are obtained through the work, can be utilized by the global optical potentials.

Keywords: Angular distribution; Oxygen-16; DWBA; NRA

1. Introduction

Ion Beam Analysis (IBA) is none destructive method that is used to probe the elemental and isotopic composition, and to determine the depth profile in the near-surface layers of solids. In this method, the near-surface layer is analyzed using ion beams with a few MeV energies. Nuclear Reaction Analysis (NRA), which is a sub-technique in IBA, is employed to perform depth profiling of the light elements (carbon, nitrogen, oxygen …) in the heavy element substrates such as metal patina [1]. Deuteron-induced reaction is an effective way in NRA due to the high positive Q-values that produces the high energy particles, and therefore these particles are easily detectable and distinguishable at the energy spectrum of all emitted particles [2]. It should be noted that, the availability and reliability of the respective differential cross sections are among the crucial factors to achieve the reliable results.

In order to determine the differential cross section and angular distribution of deuteron- induced reactions in ^{16}O, a few measurements have been made [3-5]. Besides, to interpret the experimental data, in theoretical approach some efforts have been made [6-8]. Even though the majority of available data, are differential cross sections in backward angles [9-11], a few data relating to the angular distribution are published [6, 7]. In 1973, Cavallaro measured the angular distribution of the mentioned reactions (with deuteron beams with energy below 2 MeV). He also employed the distorted wave Born approximation (DWBA) formalism to calculate the angular distribution of the stripping reaction (d,p_0). The calculation results are comparable to some extent with the measured data [7]. In addition, Gurbich calculated the differential cross sections of numerous reactions. His calculation produces the results that show a discrepancy (especially in the forward angles) between the theoretical results and experimental data [12].

In this work, we present the calculation results of the angular distribution of $^{16}O(d,p_0)^{17}O$ reaction at 1.3, 1.6 MeV energies in the lab frame. For this purpose, the DWBA formalism is employed to obtain the adjusted optical potential parameters. Moreover, the spectroscopic factors of the mentioned reaction are obtained.

2. DWBA formalism

The stripping reaction A(d,p)B is a transfer reaction which is denoted by

$$(p+n)+A \rightarrow p+(n+A) \qquad (1)$$

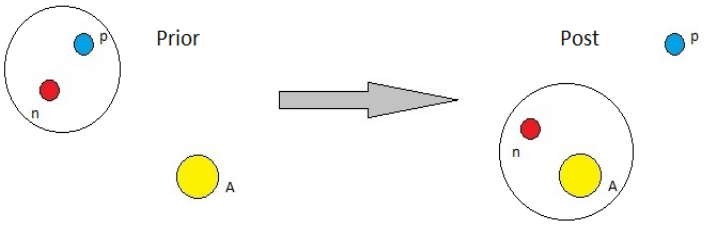

Fig. 1. The schematics of prior and post form for DWBA formalism. In each form, the bound particles are surrounded by a circle.

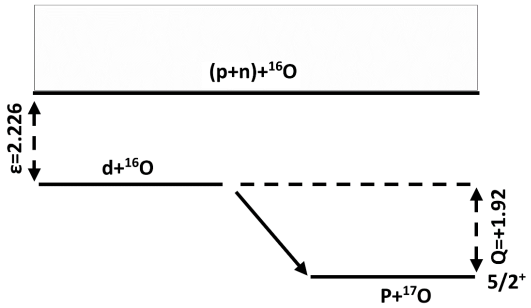

Fig. 2. Coupling scheme for the $^{16}O(d,p_0)^{17}O$ transfer reaction. The solid arrow shows the transition.

Where p represents proton, n represents neutron, d represents deuteron, A represents the target and (n+A) represents the final nucleus [13]. The stripping reaction is schematically shown in fig. 1.

If transfer couplings are weak the stripping reaction amplitude is calculated using DWBA equations (Eq. 2)

$$f_{DWBA} = A_i A_f \chi^{(-)}_{p-(A+n)} \phi_{(A+n)} | V_{prior/post} | \chi^{(+)}_{d-A} \phi_d \quad (2)$$

$$V_{Prior} = V_{n-A} + U_{p-A} - U_{d-A} \quad (3)$$

$$V_{Post} = V_{n-p} + U_{p-A} - U_{d-(A+n)} \quad (4)$$

Where f_{DWBA} is the transfer amplitude, $A_i = d|p$ (the overlap between deuteron and proton wave functions) and $A_f = (n+A)|A$ (the overlap between the target and final nucleus wave functions) are the initial and final spectroscopic amplitudes, respectively. In addition, $\chi^{(-)}_{p-(n+A)}$ and $\chi^{(+)}_{d-A}$ are the initial and final distorted waves, respectively, V_{Prior} and V_{Post} are the transition potentials in prior and post form, Φ_d is the projectile bound wave function and $\Phi_{(n+A)}$ is the final wave function. Moreover, V_{n-A}, U_{p-A}, U_{d-A}, V_{n-p}, U_{p-A} and $U_{d-(A+n)}$ represent the interaction potential between neutron and target, proton and target, deuteron and target, neutron and proton, proton and target nucleus and deuteron and final nucleus (A+n), respectively.

Finally, the differential cross section is calculated using equation

$$\frac{d\sigma}{d\Omega} = \frac{m_d m_p}{(2\pi\hbar^2)^2} \frac{\boldsymbol{K}_p}{\boldsymbol{K}_d} |f_{DWBA}|^2 \quad (5)$$

Where m_d and m_p are the reduced masses of deuteron and proton, $\hbar$ is the reduced Planck's constant, $\boldsymbol{K}_p$ and $\boldsymbol{K}_d$ are the momentum vector of proton and deuteron.

3. Results and discussion

The post form of the DWBA formalism is employed to analyze experimental angular distribution of the stripping reaction $^{16}O(d,p_0)^{17}O$ at the energy of 1.3, and 1.6 MeV in the lab frame. The schematic of the transfer reaction is shown in Fig. 2 [8]. The angular distribution is calculated using equation

$$f_{DWBA} = \langle d|p\rangle\langle (n+A)|A\rangle \left\langle \chi^{(-)}_{p-(n+A)} \Phi_{(n+A)} | V_{n-p} + U_{p-A} - U_{d-(A+n)} | \chi^{(+)}_{d-A} \Phi_d \right\rangle \quad (6)$$

It is assumed the valence neutron occupies $1d_{5/2}$ level in shell model. Computer code FRESCO is employed to calculate the angular distribution [14]. The essential physical parameters, which are used through the calculations, include the initial and final spectroscopic factors, distorted potential for the entrance channel ($d+^{16}O$), distorted potential for the exit channel ($p+^{17}O$), the core-core potential for ($p+^{16}O$), the binding potential for projectile (p+n), and the binding potential for target ($n+^{16}O$). The optical model is used for all mentioned potentials except the projectile binding potential. The form of optical potential is [13]

$$U(r) = V_c(r) - Vf_i(r) + 4ia_i W \frac{df_i(r)}{dr} + V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{dg_i(r)}{dr}(2l.s) \quad (7)$$

$$f_i(r) = \frac{1}{1+\exp(\frac{r-r_i}{a_i})},\ i = V,\ W \quad (8)$$

$$g_i(r) = \frac{1}{1+\exp(\frac{r-r_i}{a_i})},\ i = V_{so} \quad (9)$$

Where $V_c(r)$ is the coulomb term, $Vf_i(r)$ is the real volume term, $4ia_i W \frac{df_i(r)}{dr}$ is the imaginary volume

Table 1. The adjusted optical model parameters at 1.3 MeV that used in the DWBA calculations. Potential depths are in MeV and radii and diffusiveness are in fm.

parameters	Entrance	Exit	Core-Core	Binding(n+^{16}O)
V	99.77	60.1	60.1	48.74
r_V	1.013	1.26	1.26	1.25
a_V	0.879	0.65	0.65	0.65
W	7.1	8.9	8.9	6.75
r_W	1.810	1.26	1.26	1.25
a_W	0.356	0.7	0.7	0.7
V_{SO}	7.5	5.0	5.0	5.5
r_{SO}	1.45	1.25	1.25	1.25
a_{SO}	0.65	0.65	0.65	0.65
r_c	1.25	1.25	1.25	1.0

Table 2. The adjusted optical model parameters at 1.6 MeV that used in the DWBA calculations. Potential depths are in MeV and radii and diffusiveness are in fm.

parameters	Entrance	Exit	Core-Core	Binding(n+^{16}O)
V	111.10	53.5	53.5	47.5
r_V	1.010	1.25	1.25	1.25
a_V	0.870	0.65	0.65	0.65
W	6.8	7.7	7.7	7.7
r_W	1.885	1.25	1.25	1.25
a_W	0.364	0.7	0.7	0.7
V_{SO}	6.0	5.5	5.5	5.5
r_{SO}	1.4	1.25	1.25	1.25
a_{SO}	0.7	0.65	0.65	0.65
r_c	1.25	1.25	1.25	1.0

term, $V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{dg_i(r)}{dr}(2l.s)$ is the real spin-orbit term, r_i and a_i are the radii and the diffusivities, respectively, m_π is the pion mass, l is the orbital angular momentum of relative motion of the scattered particle and the nucleus and s is its spin. The coulomb term is the interaction of a point charge with uniformly charged sphere of radius R_c

$$V_c(r)=\begin{cases}\left(\frac{3}{2}-\frac{r^2}{2R_c^2}\right)\frac{Z_p\ Z_s\ e^2}{R_c} & r\le R_c\\ \frac{Z_p\ Z_s e^2}{r} & r>R_c\end{cases} \qquad (10)$$

Where Z_p and Z_s are the atomic number of the point and charged sphere, respectively.

A potential with a Gaussian geometry $(V(r)=-V_0\exp[-(r/r_0)^2])$ with parameters V_0=72.15 MeV and r_0=1.484 fm represents the projectile binding potential [15]. The potential parameters of the entrance channel potential are obtained using the energy-dependent global optical potential from Ref. [16]. In addition, the energy-dependence potential parameters are employed for the exit channel, the target biding potential, and the core-core potential from Ref. [17]. Moreover, the average compound nuclease (CN) contribution at energies 1.3 and 1.6 MeV are calculated 1.3 and 1.8 mb, respectively [7]. Due to numerous variable parameters, it is assumed that some of them, which have less effect on the final results, are fixed. To fit the other parameters with experimental data from Ref. [7], first the compound nucleus contributions are reduced form the experimental data, then the fitting is conducted to obtain the adjusted potential parameters. Moreover, the initial spectroscopic factors $(S_i=|A_i|^2=|\langle d|p\rangle|^2=1)$ and the final spectroscopic factor $(S_f=|A_f|^2=|\langle(n+A)|A\rangle|^2=0.83)$ are obtained. All adjusted optical parameters, which are written in bold number, along with the rest are listed in Table 1, and 2. In addition, the calculated and experimental angular distribution are shown in Fig. 3, and 4.

If one contracts the obtained results with the calculations that made by Cavallaro and Gurbich, It is clear that the angular distribution, which are produced by our adjusted parameters, is better fitted with the experimental data. It shows that the assumption ($1d_{5/2}$ level) about the valence neutron in ^{17}O is correct. Cavallaro performed the same procedure to obtain theoretical angular distribution but he did not report his assumption about the valence neutron and he also did not manage to adjust the potential parameters. Moreover, he ignored the spin-orbit term for optical potential. Besides, Gurbich employs R-Matrix theory to calculate compound nucleus (resonance) contribution and used optical model to calculate the direct reaction (stripping reaction) contribution. The direct reaction ^{16}O(d,p$_0$)^{17}O is a transfer reaction, which target nucleus ^{16}O ($I^\pi = 0^+$) converts to the ground state ^{17}O ($I^\pi = 5/2^+$ E_x=0). In this case, the angular distribution must be comparable to the experimental data while the graph, which is produced by Gurbich, is comparable with the transition to the first excited state of ($I^\pi = 1/2^+$ E_x=0.871). The reason for discrepancy (especially in the forward angles) is, he concentrated on the backward angles where it is more applicable in IBA.

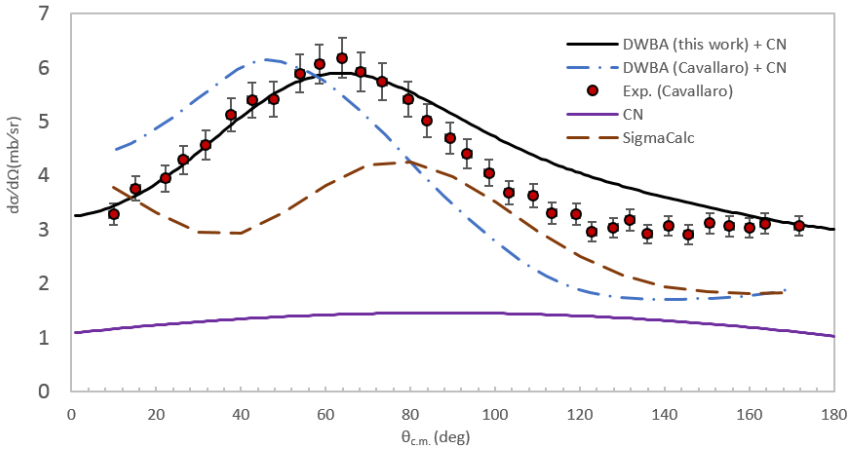

Fig. 3. The angular distribution of $^{16}O(d,p_0)^{17}O$ at 1.3 MeV for the present work along with experimental and theoretical results from the Ref. [7], compound-nuclear (CN) contribution, and the calculation of Ref. [12].

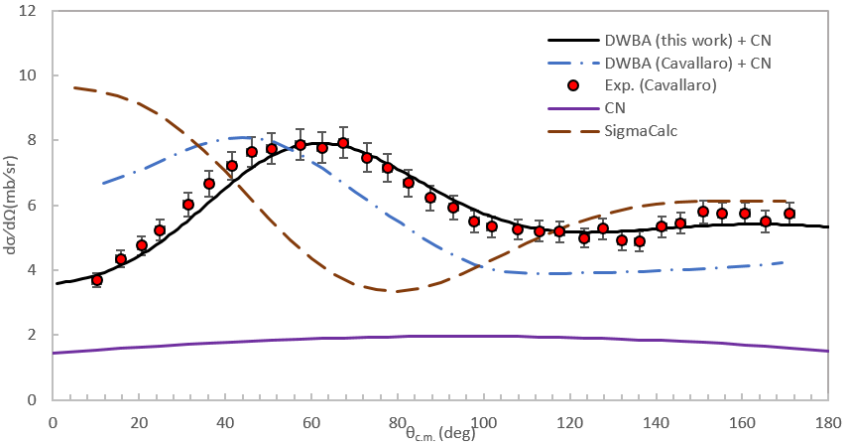

Fig. 4. The angular distribution of $^{16}O(d,p_0)^{17}O$ at 1.6 MeV for the present work along with experimental and theoretical results from the Ref. [7], compound-nuclear (CN) contribution, and the calculation of Ref. [12].

1. Conclusion

In this work, the DWBA formalism is employed to calculate the angular distribution of the transfer reaction (stripping reaction) $^{16}O(d,p_0)^{17}O$ at 1.3, and 1.6 MeV energies in the lab frame. The adjusted optical model parameters and the spectroscopic factors, which are obtained through the calculation, produce the results that are better fitted to the experimental data in comparison with the previous works. The global potential provides the optical potential parameters for a specific reaction in a wide range of the incident energies. To develop a global potential, it is essential to have the optical potential parameters in some steps in the range. As the previous calculations show inconsistencies at the energies below 2 MeV, our data can be utilized to modify the global potential. Moreover, as IBA needs the differential cross sections at the different angles, the calculated cross sections can be also useful for this purpose.

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