Hypercentral constituent quark model and the hyperfine dependence potential

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Abstract

In this article nucleons are discussed based on constituent quark model. This model aims at studying the forces among three particles and the corresponding standard two-body potential contribution. The quark potential contains a hypercentral interaction. The confining potential is composed of four components: color charge, the oscillatory potential, the interaction quark and neutral gluon, and the dipole – dipole electromagnetic interaction. Dirac equation can be solved carefully and analytically by means of these potentials. In addition to the above potentials, there is a hyperfine potential which is related to isospin – isospin and spin –isopin interactions. These potentials were considered as perturbation potentials and their energy shift was calculated.

Keywords: Dirac equation, hypercentral, Jacobian coordinate, hyperfine-isospin

1. Introduction

Nucleons are usually described quite well by different Constituent Quark Models (CQM). Constituent Quark Models have been widely used recently for investigation of nucleon structure. There are different methods for studying the nucleons. Although many different potentials for quarks have been studied but the study of the structure of nucleons is not sufficient. In this paper we take the potential between quarks as hypercentral. This potential has four subcomponents.

The first potential is the corresponding short range interaction which has an attractive hyper coulombic potential ,can be considered as $-\dfrac{c}{x}$. [1,2,3] it originates from the interaction of color charge. In this formula x is the hyperradius defined in terms of quarks interaction [4, 5, 6, 7]:

$$x = \sqrt{\rho^2 + \lambda^2} \ , \qquad \xi = tg^{-1}(\frac{\rho}{\lambda}) \ , \qquad (1)$$

ρ and λ are Jacobin coordinates and ξ is hyperangle, together with the angles Ω_ρ, Ω_λ while at large separations the hyperoscillator potential which has a two-body character turns out to be hypercentral. This potential can be formulated as follows.

$$V_{h.0} = \sum_{i<j}^{i=3} \frac{1}{2} k \, (r_i - r_j)^2 = \frac{3}{2} k \, x^2 = a \, x^2 \ . \qquad (2)$$

These terms show the oscillation of one quark relative to the other quarks [8, 9, 10, 11, 12].

The other potential is the interaction of one quark with a neutral gluon. This interaction can be shown as $v (x) = \dfrac{d}{x^4}$, the origin of which is the electrical charge.

The last potential is the interaction of one color magnetic filed with the external dipole field of the other interaction containing the spin-spin.

$$v(x) = \sum_{i,j} \frac{2\alpha_s}{r^3} (\frac{3\vec{s_i}.\vec{s_j}.\vec{r}}{r^2} - \vec{s_i}.\vec{s_j}) = \frac{c_f \alpha_s s_{12}}{m^2 x^3} \ . \qquad (3)$$

This formula can be simplified as: $v(x) = \dfrac{b}{x^3}$ where

$b = \dfrac{c_f \, \alpha_s \, s_{12}}{m^2}$ and s_{12} is tensor operator.

Extra hyperfine dependent quark potential which contains isospin-isospin dependent $H_I(x)$ and spin – isospin dependent $H_{SI}(x)$ are also important. The interaction will be complete by adding the following components.

$$H(x) = v(x) + H_i(x) + H_{SI}(x) \ . \qquad (4)$$

In this formulation $v(x)$ is the confining potential. The interaction hyperfine potential isospin is more important in the quark interaction.

The aim of this paper is threefold: adding $v(x)$ to the

Dirac equation, providing an exact solution, and finally considering the hyperfine interactions as perturbation potential thereby calculating first order energy shift.

2. Dirac equation with hypercentral potential

The Dirac equation may transform in various ways under a Lorentz transformation. The form in common use for scalar hypercentral potential (u_0) and vector hypercenteral (v_0) is:

$$(\sigma.p)\chi + (m + u_0 + v_0)\varphi = \varepsilon\varphi \quad , \tag{5-a}$$

$$(\sigma.p)\varphi - (m + u_0 - v_0)\chi = \varepsilon\chi \quad . \tag{5-b}$$

The internal quark motion is described by Jacobian coordinates:

$$\rho = \frac{\vec{r}_2 - \vec{r}_1}{\sqrt{2}} \quad , \qquad \lambda = \frac{\vec{r}_1 + \vec{r}_2 - \vec{r}_3}{\sqrt{6}} \quad . \tag{6}$$

By using the Jacobin coordinate for ρ, λ the Dirac equation for the λ component is as follows:

$$(\sigma.p_\lambda)\chi + (m + u_0 + v_0)\varphi = \varepsilon\varphi \quad , \tag{7-a}$$

$$(\sigma.p_\lambda)\varphi - (m + u_0 - v_0)\chi = \varepsilon\chi \quad . \tag{7-b}$$

In the introduction section of the article we assumed four potentials. Now in the foregoing equations we can replace $v_0(x)$ with the defined components [13,14,15]:

$$2u_0(x) = 2v_0(x) = (ax^2 - \frac{c}{x} + \frac{b}{x^3} + \frac{d}{x^4}) \quad .$$

Now by combining (7-a) and (7-b), we obtain:

$$p_\lambda{}^2\varphi + (m^2 - \varepsilon^2)\varphi + 2u_0(m + \varepsilon)\varphi = 0 \quad . \tag{8}$$

To avoid repletion, it should be noted that the same calculations can be applied to ρ component.

$$p_\rho{}^2\varphi + (m^2 - \varepsilon^2)\varphi + 2u_0(m + \varepsilon)\varphi = 0 \quad . \tag{9}$$

By adding (8) and (9), we can get to:

$$(p_\rho{}^2 + p^2\lambda)\varphi + 2(m^2 - \varepsilon^2)\varphi + 4u_0(m + \varepsilon)\varphi = 0 \quad . \tag{10}$$

For the upper component, the wave function can be taken as follows:

$$\psi_{p=0}(p_\rho, p_\lambda) = u(p_1)u(p_2)u(p_3)\varphi(p_\rho, p_\lambda) \quad , \tag{11}$$

where $\varphi(p_\rho, p_\lambda)$ is the eigenfunction and the parenthetical operators can be obtained from:

$p_\rho = \frac{p_1 - p_2}{\sqrt{2}}$ and $p_\lambda = \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}$, and where

$u(p_i)$ is the Dirac spinor of the i-th quark and p_i is momentum of the i-th quark in the nucleon rest frame.

The internal quark motion is usually described by means of the Jacobi relative coordinates. By separating the common motion, the P^2 operator of a quark in the 3q system becomes ($\hbar = c = 1$) [2].

$$(\Delta^2{}_\rho + \Delta^2{}_\lambda) = (\frac{\partial^2}{\partial x^2} + \frac{5}{x} + \frac{L^2(\Omega)}{x^2}) \quad . \tag{12}$$

The eigenvalues of $L^2(\Omega)$ are given $L^2(\Omega) = -\gamma(\gamma + 4)$ where γ is the grand angular quantum number and can be given by $\gamma = 2n + l_\rho + l_\lambda$. In this relation l_ρ, l_λ are the angular moments associated with the ρ and λ variable. Considering the defined potentials, (10) can be embedded in (12) and the result will be:

$$-\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{L^2}{x^2}\right]\varphi = \left[2(\varepsilon^2 - m^2) - (\varepsilon + m)(ax^2 - \frac{c}{x} + \frac{b}{x^3} + \frac{d}{x^4})\right]\varphi. \tag{13}$$

To simplify the above equation, we can replace the constants as follows:

$$E = 2(\varepsilon^2 - m^2) \quad , \quad A_1 = (\varepsilon + m)a, B_1 = (\varepsilon + m)b, \\ C_1 = (\varepsilon + m)c, D_1 = (\varepsilon + m)d \quad . \tag{14}$$

With the foregoing assumptions at hand, eq. (13) can be simplified as:

$$\varphi''(x) + \frac{5}{x}\varphi' + \left[E - A_1x^2 + \frac{C_1}{x} - \frac{B_1}{x^3} - \frac{D_1}{x^4} - \frac{\gamma(\gamma + 4)}{x^2}\right]\varphi = 0 \quad . \tag{15}$$

This equation is too difficult to solve. However the author found an ansatz solution [16, 17].

$$\varphi(x) = h(x)\exp[y(x)] \quad . \tag{16}$$

We logically propose $h(x)$ and $y(x)$ as functions the coefficients of which should be determined [18, 19].

$$h(x) = 1 \quad , \qquad y(x) = \frac{-1}{2}\alpha x^2 - \frac{\beta}{x} + \delta Ln(x) \quad . \tag{17}$$

We put the answer to eq. (16) in eq. (15). From the equality of the coefficients we have:

$$\beta = \gamma \quad , \quad \alpha = \frac{(\varepsilon^2 - m^2)}{2(3 + \gamma)} \quad , \quad \delta = \frac{(\gamma + 3)}{(\varepsilon - m)}c \quad . \tag{18}$$

Taking this answer, now we can obtain the coefficients in eq. (14).

$$A_1 = \frac{(\varepsilon^2 - m^2)}{4(3 + \gamma)^2}, B_1 = \frac{(2\gamma + 3)(\gamma + 3)c_1}{\varepsilon^2 - m^2}, D_1 = \frac{(3 + \gamma)^2 c_1^2}{(\varepsilon^2 - m^2)^2}, \tag{19}$$

with the obtained coefficients, we can now obtain the upper component as follows:

$$\varphi(x) = x^2 \exp(-\frac{1}{4}\frac{(\varepsilon^2 - m^2)}{(3 + \gamma)}x^2 - \frac{(\gamma + 3)c}{(\varepsilon - m)x}) \quad . \tag{20}$$

The lower component χ of the Dirac hyper-central spinor can be derived from eq. (5) χ can be derived from eq. (5-b) as follows:

$$\chi = = \frac{(\sigma.p_\lambda)\varphi}{(m + \varepsilon)} \quad , \quad \chi = = \frac{(\sigma.p_\rho)\varphi}{(m + \varepsilon)} \quad . \tag{21}$$

If we add these two equations and take $\vec{p} = \vec{p}_\lambda + \vec{p}_\rho$, then:

$$\chi == \frac{(\vec{\sigma}.\vec{p})\varphi}{(m+\varepsilon)} . \tag{22}$$

Based on the upper and lower components the wave functions for grand state, i.e., $\gamma = l = 0$ would be as follows:

$$\psi(x) = N \begin{bmatrix} \exp(-\frac{1}{2}\alpha x^2 - \frac{\delta}{x}) \\ \frac{-i\vec{\sigma}.x}{2(\varepsilon+m)} \frac{d}{dx} (\exp(-\frac{1}{2}\alpha x^2 - \frac{\delta}{x}) \end{bmatrix} . \tag{23}$$

For a closer study of the structure of nuclons, the potentials spin-isospin should also be taken into account. In what follows we will introduce these potentials. This will help us to calculate the energy shifts based on perturbation theories.

3. The effect of spin-isospin interaction in structure of nucleon

There are different motivations for the introduction of a residual flavor dependent term in the three quark interaction. The standard hyperfine interaction is used to reproduce the splitting within the SU (6) multiplets. It contains a δ-like term which is an illegal operator [20]:

$$H_{Spin} = B_S \left(\frac{1}{\Lambda_S} e^{-(\frac{x}{\Lambda_S})} \right) \sum_{i<j}^{i=3} (\vec{s}_i.\vec{s}_j) , \tag{24}$$

where s_i is the spin operator of the i-th quark and $x = r_{ij}$ is the relative quark pair coordinate. The strength of the hyperfine interaction is determined by the $\Delta - N$ mass difference. The fitted parameters are :

$B_s = 1964 \ fm^2$, $\Lambda_s = 1.6$ fm.

To introduce the isospin nonconfing potential we have the Chiral Constituent Quark Model (CQM) [21,22]. The nonconfining part of the potential is provided by the interaction with the Goldstone bosons, giving rise to a spin- and isospin dependent part, which is crucial for the description of the spectrum for energies lower than 1.7 GeV. It has also been pointed out quite recently that an isospin dependence of the quark potential can be obtained by means of quark exchange. More generally, one can expect that the quark-antiquark pair production can lead to an effective quark interaction containing an isospin (or flavour) - dependent term. On the other hand, the fact that the constituent quark model does not explicitly contain this mechanism, may account for the low Q^2 behaviour of the electromagnetic transition in which form factors are not reproduced. With these considerations, we have introduced isospin-dependent terms in the hCQM Hamiltonian. To this end we have added two terms in the three-quark Hamiltonian with the hyperfine interaction.

The first one depends on the isospin-isospin and has the form:

$$H_I = A_I \sum_{i<j} \frac{1}{(\sqrt{\pi}\sigma_I)} e^{-(\frac{r_{ij}^2}{\sigma_I^2})} (I_i.I_j) , \tag{25}$$

where I_i is the isospin operator of the quark and $x = r_{ij}$ is the relative quark pair coordinates and $A_I = 51.7 \ fm^2$, $\sigma_I = 3.45 \ fm$

Based on the perturbation theory for the grand state $\ell = 0$, first order shift energy can be shown as : Δ_I^1 and the value of which was calculated.

$$\Delta_I = -27.62 (MeV) \qquad for \qquad I = \frac{1}{2}$$

$$\Delta_I = 27.62 (MeV) \qquad for \qquad I = \frac{3}{2} \tag{26}$$

The second potential is a spin-isospin interaction, given by

$$H_{sI} = A_{sI} \sum_{i<j} \frac{1}{(\sqrt{\pi}\sigma_{sI})} e^{-(\frac{r_{ij}^2}{\sigma_{sI}^2})} (s_i.s_j)(I_i.I_j) \tag{27}$$

Where s_i and I_i are respectively the spin and isospin operators of the $i-th$ quark and $x = r_{ij}$ is the relative quark pair coordinate and $A_{SI} = -106.2 \ fm^2$ $\sigma_{SI} = 2.31 \ fm$ [22].

The first order energy shift can be shown as : Δ_{SI}^1 and the value of which is as follows:

$$\Delta_{si} = -13.4(MeV) \qquad for \qquad I = \frac{1}{2} \ and \ s = \frac{1}{2} . \tag{28}$$

In this paper we consider the completely interaction between quarks in a nucleon including spin and isospin interaction. The results are applied to all of the baryons as well. The higher-order correction will give better results. Since this model gives reasonable results, it would lead us to determine the kind of modification which yield the observable static properties of a nucleon that is super singly close to the experiment and we can determine the form factors of nucleons and root mean square radius (RMS)

4. Conclusion

An exact analytical solution for potential in the form of the confinement is presented. The hypercenteral potential is a good starting point for investigation of nucleon structure

A considerable improvement in the description of the static properties of nucleon is obtained with an isospin-dependent potential. In fact in this model the energy shift comes from the spin –isospin term is considerable. By Using this model we can investigate the other baryons.

The splitting which are in general spin and isospin dependent, can be treated pertubatively. Finally one can use this model and determine the mass of baryons.

References

1. M Bohm, *Z. Phys*. C **3** (1980) 321: Im, Barbour and D K ponting, *Z. Phys*. C **4** (1980) 119; A M Badalyan, *Phys. Lett*. B **199** (1987) 267; M Fabre de la Ripelle, *Nucl. Phys*. A **497** (1989) 595.
2. M M Giannini, E Santopinto and A Vassallo, *Nuclear Physics* A **699** (2002) 308-311.
3. A A Rajabi, *Indian Journal of Pure and Applied Physics* **41** (2003) 89-94.
4. L Glozman, ya D O Riska, *Phys, Rep*. C **268** (1996) 263.
5. J L Richardson, *Phys. Lett*. B **82** (1979) 272.
6. W Buchmuller and S-H H Tye, *Phys. Rev*. D **24** (1981) 132.
7. V A Nowikoy, L B Okun, M A Shifman, A I Vainshtein, M B Voloshin and V I Zakharov, *Phys. Rep*. C **41** (1978) 1; S Yazaki, *Nucl. Phys*. B **186** (1981) 109.
8. L J Reinders, H R Rubinstein and S Yazaki, *Nucl. Phys*. B **186** (1981) 109.
9. K Gottfried, "Hadronic Spectroscopy", in: Proc. Int. Euro: *Phys. Conf. on High Energy Physics*, Brighton, (1983).
10. L J Reinders, H R Rubinstein and S Yazaki, *Phys. Rep*. C **127** (1985) 1.
11. M Campostrini, K Moriarty, C Rebbi, *Phys. ReV*. D **36** (1987) 3450; L Heller, in quarks and nuclear forces, edited by D C Vries, B Zeitnitz, Springer Tracts, *Mod. Phys*., **100** (1982) 145.
12. S Bali Gunnar, et al., *Phys. Rev*. D **62** (2000) 054503; S Bali Gunnar, *Phys. Rep*. **343** (2001) 1.
13. A A Rajabi, *Commun Theor. Phys*. (Bijing, China) **45,** 4 (2006) 15.
14. G B Smith and J Tassie: *J. Phys*. Ny **65** (1971) 352.
15. J S Bell and H Ruegg: *Nucl. Phys*. B **98** (1975) 151; J S and H Ruegg, *Nucl. Phys*. B **104** (1976) 546.
16. M Znojil, *J. Math Phys*. **31** (1990).
17. M Znojil, *J. Math Phys*. **30** (1989) 23; M J Znojil, *Math Phys*. A **15** (1982) 2111.
18. R S Kaushal: *Ann Phys* Ny **206** (1991) 90.
19. S Oezelik and M Simek: *Phys. Lett*. A **152** (1991) 145.
20. R S Kaushal, *Phys. Lett*. A **142** (1989) 57.
21. A A Rajabi, *Iranian Journal of Physics Research*, **6** No. 2 (2006).
22. M M Giannini, E Santopinto and A Vassallo, *Eur. Phy. J*. A **12** (2001) 447-452.