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Research note

Calculating charge radius for proton with hyper central interacting color potential

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Abstract
An improved M.I.T. bag model with hyper central interaction is used to calculate the charge radius for proton containing u and d quarks. We present a theoretical approach to the internal structure of three-body hyper central interacting quarks in a proton, in which we take proton as a bag. We discuss a few of results obtained using a six-dimension potential, which is attractive for small separation, originating from the color charge of hyper color term. We calculate the relativistic wave function for quarks in a scalar-vector hyper central potential, analytically. Finally, vanishing the normal component of vector current at the surface of the proton bag as a boundary condition equivalent to confinement. The calculated charge radius for proton is better than in the uncorrected versions of the model.

Keywords: Dirac equation, hyper central interaction, color potential, vector current, charge radius

1. Introduction

In the MIT bag model, hadrons are considered as extended static objects localized in space in which each quark moves freely in a spherically symmetric well of infinite depth (bag). The internal structure of particles is associated with quark and gluon field variables. Because of its simplicity, the MIT bag model [1] is rather convenient for calculating various hadronic properties. The MIT bag model possesses many desirable features inspired by QCD and relativity. However, so far, there is no derivation of the bag model from first principles. Thus, it is important to test the model in a situation other than those in which the dynamics of the model were originally formulated. The success of the first attempts to calculate the static properties of hadrons lend credence to the bag model approach to hadronic phenomenology. By using the normalized spin 1/2 positive parity solution of MIT bag model for studying nuclear matter saturation mechanism, based on the quark structure of the nucleon [2] and also recently, a quark-meson coupling QMC mechanism for the saturation of nuclear was initially proposed by Guichon [3] and generalized by Fleck *et al.* [4], Saito & Thomas [5] and Song & Su [6]. In the QMC model, the baryon is described by the static spherical MIT bag model, using the normalized ground-state for quark in the baryons.

Although the MIT bag model gives results, which are within an acceptable range that shows the correctness of its essential ingredients, obviously, has short comings.

One of the short coming is the neglecting of inter-quark interactions. The MIT bag model user free quark wave function with no-current boundary condition in the bag' s wall. Certainly, quarks interact within the bag, which will change the standard results. In the past, giving rise to better results compared to experimental values [7, 8].

In this paper, a modification is proposed by extending the model to include certain residual hyper central interaction among quarks. In our model, each quark moves inside the bag in the effective field of the other quarks. The relativistic Dirac equation is considered, but with the above mentioned effective hyper central potential.

Now the interaction that is considered is briefly discussed. The effective interaction of quarks due to gluon exchange is assumed to be given by hyper central potential for each quark. It is assumed that the internal quark motion is described by the Jacobi coordinates ρ and λ [9, 10]. In order to describe the three quark dynamics it is convenient to introduce the hyper spherical coordinates, which are obtained by substituting the absolute values of ρ and λ in $x=\sqrt{\rho^2+\lambda^2}$, where x is the hyper radius. The potential is the six-dimensional hyper Coulomb potential [10, 11], which is attractive for small separation originating from the color charge:

$$V_{hyc}(x)=-\frac{c}{x}. \tag{1}$$

In section 2 we have calculate the relativistic wave function for valence quarks. The potential parameter can be found by fitting as in section 3. The results indicate that this potential is useful for quarks having masses in the range used in the phenomenological analysis of the quark model and finally in section 4, we calculate charge radius for proton. In our model it is concluded that there is a reasonable consistency between the calculated values and the experimental results.

In the following section, by solving the Dirac equation, the relativistic wave function for valence quarks has been calculated analytically. The proton masses and the ratio $\frac{g_A}{g_V}=1.254\pm0.006$ [12] are taken as our inputs. These inputs fix the parameter in potential and the quark mass, and from them charge radius for proton is derived. The numerical values and charge radius of our model with three quarks potential, suitable for constituent quark's masses, which are in the range of (100-350) Mev show remarkable improvement over previous results obtained by the MIT bag model quarks potential.

2. Hyper central relativistic wave function for three quarks in proton

The constituent quark model based on a hyper central approach takes into account three body force effects and the standard two-body potential contributions. Let's represent the quark wave function satisfying the Dirac equation by $\psi(\vec{x})$, so

$$[\gamma_0\varepsilon+i\vec{\gamma}.\vec{\nabla}-(m+U(x))]\psi(\vec{x})=0. \tag{2}$$

The hyper central potentials, which lead to analytical solution in our model, would be

$$U(x)=\frac{1}{2}(1+e\gamma_0)A(x). \tag{3}$$

The parameter e can take any value [13-15]. In this investigation it is taken as 1. Hence, from eq. (1) the interaction potential can be taken as

$$A(x)=-\frac{c}{x}. \tag{4}$$

The quark potential, $U(x)$, is assumed to depend on the hyper radius x only. The Dirac equation may transform in various ways under a Lorentz transformation. The form in common use for Scalar Hyper Central Potential ($U_0(x)$) and vector Hyper Central Potential ($V_0(x)$) is often taken as follows:

$$\begin{aligned}&-i\vec{\alpha}.\vec{\nabla}\psi(x)+\beta[m+U_0(x)]\psi(x)\\&+(V_0(x)-\varepsilon)\psi(x)=0\end{aligned} \tag{5}$$

From eqs. (2, 3 and 4)

$$V_0(x)=U_0(x)=\frac{1}{2}A(x). \tag{6}$$

For any quark, the eigenspinor of (4) is rewritten as

$$\begin{cases}(\vec{\sigma}_i.\vec{P}_i)\chi_i+(m_i+U_0(x)+V_0(x))\phi_i=\varepsilon_i\phi_i,\\(\vec{\sigma}_i.\vec{P}_i)\phi_i-(m_i+U_0(x)-V_0(x))\chi_i=\varepsilon_i\chi_i.\end{cases} \tag{7}$$

By sum over the eqs. (7) and combine with two equations (7) for the Dirac upper component and from eqs. (6, 7) we have

$$\begin{aligned}(P_1^2+P_2^2+P_3^2)\phi&=9(\varepsilon^2-m^2)\phi\\&-6\,V_0(\varepsilon+m)\phi,\end{aligned} \tag{8}$$

here

$$\begin{cases}\phi=g_\gamma(x)Y_{jl}^{j_3}(\hat{x}),\\\chi=if_\gamma(x)Y_{jl'}^{j'_3}(\hat{x}).\end{cases} \tag{9}$$

The internal quarks motion is usually described by means of the Jacobi relative coordinates. After separating the common motion, the P^2 operator of a quark in the 3q system becomes ($\hbar=c=1$) [10]

$$P^2=-(\nabla_\rho^2+\nabla_\lambda^2)=-(\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}+\frac{L^2(\Omega)}{x^2}), \tag{10}$$

hence

$$\begin{aligned}&(\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}+\frac{L^2(\Omega)}{x^2})\phi\\&+3[3(\varepsilon^2-m^2)-A(x)(\varepsilon+m)]\phi=0,\end{aligned} \tag{11}$$

with $A(x)$ given by (4), where $L^2(\Omega)=-\gamma(\gamma+4)$ is the grand orbital operator and γ is the grand angular quantum number given by $\gamma=2n+l_\rho+l_\lambda$. First of all the transformation

$$\phi_\gamma=x^{-\frac{5}{2}}\xi_\gamma(x), \tag{12}$$

reduces (11) to the form

$$\begin{aligned}&\frac{d^2\xi_\gamma(x)}{dx^2}-\frac{15}{4x^2}\xi_\gamma(x)+\frac{L^2(\Omega)}{x^2}\xi_\gamma(x)\\&+3[3(\varepsilon^2-m^2)-(\varepsilon+m)A(x)]\xi_\gamma(x)=0.\end{aligned} \tag{13}$$

Now, for the eigenfunction $\xi_\gamma(x)$ we make an ansatz [11, 16-19]

$$\xi_\gamma(x)=h(x)\exp[z(x)], \tag{14}$$

with $h(x)$ and $z(x)$ given by

$$\begin{cases}h(x)=1+\alpha x+\kappa x^2,\\z(x)=-\beta x+\delta\ln(x).\end{cases} \tag{15}$$

This implies

$$\begin{aligned}\xi''_\gamma(x)+\frac{15}{4}\xi_\gamma(x)&=(z''(x)+(z'(x))^2\\&+\frac{h''(x)+2h'(x)z'(x)}{h(x)}-\frac{15}{4x^2})\xi_\gamma(x).\end{aligned} \tag{16}$$

Table 1. Comparing static properties of proton in our model and MIT bag model with experiments.

Proton	MIT bag model	Our model	Experiment
m_q	0	320.15 ± 0.01 (Mev)	~100~350 (Mev)*
g_A/g_V	1.09	1.254 ± 0.006	1.254 ± 0.006
x_b	1.5 (fm)	1.66 ± 0.08(fm)	………
$\langle r_{em}^2 \rangle^{\frac{1}{2}}$	0.73 (fm)	0.895 ± 0.003(fm)	0.88 ± 0.03(fm)

* :The values are not directly measured but inferred from experiment

Then our purpose is to find the fraction of the power of x to the one on the left hand side of eq. (13) corresponding to the potential and energy. First of all let' s put $\gamma = 0$ for ground state. On comparing eqs. (16) and (13) we obtain

$$\begin{cases} \delta = -\dfrac{3}{2} \ and \ \dfrac{5}{2}, \\ \alpha = -\dfrac{4c(\varepsilon + m)}{15}, \\ \beta = \dfrac{c(\varepsilon + m)}{3}, \\ \kappa = -\dfrac{c(\varepsilon + m)\alpha}{18}. \end{cases} \quad (17)$$

Taking $\delta = -\frac{3}{2}$ leads to the wave function, this is well behaved at the origin.

eqs. (14, 15, 17) are used to find the upper component of the Dirac hyper central spinor $g_0(x)$ for proton with mass M as follows:

$$g_0(x) = (1 - 0.260 c_1 x + 0.014 c_1^2 x^2) \exp(-\frac{c_1}{3} x), \quad (18)$$

where $c_1 = c(\varepsilon + m)$.

The lower component $f_0(x)$ of the Dirac hyper central spinor can be found from (7). The normalized spin $\frac{1}{2}$ positive parity solution of the quarks under standard hyper spherical potential (3, 4) is introduced by the following form:

$$\psi(x) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} 1 - 0.260 c_1 x + 0.014 c_1^2 x^2 \\ \dfrac{i\vec{\sigma}.\hat{x} c_1}{(\varepsilon + m)} (0.600 - 0.118 c_1 x \\ + 0.004 c_1^2 x^2) \end{pmatrix} \exp(-\frac{c_1}{3} x). \quad (19)$$

The wave function is different from standard MIT bag wave function. From eq. (19), the bag radius x_b is determined by solving the boundary equation. This shows that the normal component of vector current vanishes at the surface of the proton bag, just like as in MIT bag model [1]

$$\begin{aligned} &(0.014 - 0.004c) c_1^2 x_b^2 \\ &+ (0.118c - 0.260) c_1 x_b + 1 - 0.600c = 0. \end{aligned} \quad (20)$$

The bag radius (x_b) is determined by solving the above equation.

3. Ratio of g_A to g_V for proton

In this section, it is explained how the ratio g_A/g_V is used as an input. The ratio of axial vector coupling constant g_A to vector coupling constant g_V in the relativistic case satisfies [1]

$$\frac{g_A}{g_V} = \frac{5}{3}(1 - 2\langle l_z \rangle) = \frac{5}{3}(1 - 2\langle \psi_\gamma | l_z | \psi_\gamma \rangle). \quad (21)$$

eq. (21) contains unknown parameters c and the proton mass M. In order to find the parameter c_1 for proton (M = 938 Mev), g_A/g_V can be taken as 1.254 ± 0.006 (or $\langle l_z \rangle$=0.123 ± 0.002). This value is measured experimentally [12], so we get c_1 = 5.36 ± 0.01 (fm^{-1}) and from eqs. (17), the parameter of the potential c can be found. Now from this we can get m_q = (320.15 ± 0.01) Mev. This result is well within the expected rang.

4. Proton charge- radius

The mean-square charge radius for proton $\langle r_{em}^2 \rangle$ is defined as:

$$\langle r_{em}^2 \rangle = \sum e_q \langle r^2 \rangle, \quad (22)$$

where

$$\langle r^2 \rangle = \int_{bag} r^2 \psi^*(\vec{r}) \psi(\vec{r}) d^3 r. \quad (23)$$

By using the upper and lower components of the spinor (19) and m_q = (320.15 ± 0.01) Mev from the above, the charge-radius of proton eq. (23) we obtained $<r_{em}^2>^{1/2}$ = 0.895 ± 0.003 fm. This result is closer to the observed value 0.88 ± 0.03 fm than the previous one of the MIT bag model by 14%.

From eq. (20) and c_1 = 5.36 ± 0.01 fm^{-1} the bag radius for proton is x_b = 1.66 ± 0.08 fm.

5. Conclusions

We present a theoretical approach to the internal structure for three valance quarks in a bag with residual hyper central interaction, among them are asymptotic freedom and confinement. Hence, the results have been improved from the MIT bag model to get better results than many of the improved bag models to a large extent. From table 1 the charge radius for proton is seen to be very close to the experimental results.

This method, just like MIT bag model, can be applied to a wide variety of baryons and mesons. In general we have considered the proton only; however, in all cases our results are much closer to the experimental values than the other improved bag models.

Comparing Table 1 with MIT bag model and experimental results shows that our model certainly improves the MIT bag model, and hence, can also improve the models in references [2, 3].

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