

## The role of electroweak penguin and magnetic dipole QCD penguin on hadronic $b$ Quark Decays

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### Abstract

This research, works with the effective Hamiltonian and the quark model. Using, the decay rates of matter-antimatter of  $b$  quark was investigated. We described the effective Hamiltonian theory which was applied to the calculation of current-current ( $Q_{1,2}$ ), QCD penguin ( $Q_{3,\dots,6}$ ), magnetic dipole ( $Q_8$ ) and electroweak penguin ( $Q'_{7,\dots,10}$ ) decay rates. The gluonic penguin structure of hadronic  $b$  decays  $b \rightarrow q_k g \rightarrow q_k q_i q_{\bar{j}}$  was studied through the Wilson coefficients of the effective Hamiltonian. The branching ratios of the Tree-Level, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole  $b$  quark decays  $b \rightarrow q_i q_k q_{\bar{j}}$ ,  $q_i \in \{u, c\}$ ,  $q_k \in \{d, s\}$ ,  $q_{\bar{j}} \in \{\bar{u}, \bar{c}\}$  have been calculated. It was shown that, the electroweak penguin and magnetic dipole contributions in  $b$  quark decays are small and current-current operators are dominated.

**Keywords:**  $b$  quark, QCD penguin, electroweak penguin, magnetic dipole

### 1. Introduction

Conservation of the gluonic current requires the  $b \rightarrow q_k g$  vertex to have the structure [1,2]:

$$\Gamma_{\mu}^a(q^2) = (ig_s / 4\pi^2) \bar{u}_k(p_k) T^a V_{\mu}(q^2) u_b(p_b), \quad (1)$$

where ,

$$V_{\mu}(q^2) = (q^2 g_{\mu\nu} - q_{\mu} q_{\nu}) \gamma^{\nu} [F_1^L(q^2) P_L + F_1^R(q^2) P_R] + i\sigma_{\mu\nu} q^{\nu} [F_2^L(q^2) P_L + F_2^R(q^2) P_R]. \quad (2)$$

Here  $F_1$  and  $F_2$  are respectively the electric (monopole) and magnetic (dipole) form factors,  $q = q_g = p_b - p_k$  is the gluon four momentum,  $P_{L(R)} \equiv (1 \mp \gamma_5) / 2$  are the chirality projection operators and  $T^a$  ( $a = 1, \dots, 8$ ) are the  $SU(3)_c$  generators normalized to  $Tr(T^a T^b) = \delta^{ab} / 2$ .

The  $\bar{b} \rightarrow \bar{q}_k g$  vertex is

$$\bar{\Gamma}_{\mu}^a(q^2) = -(ig_s / 4\pi^2) \bar{v}_b(p_b) T^a \bar{V}_{\mu}(q^2) v_k(p_k) \quad (3)$$

Here  $\bar{V}_{\mu}$  has the form eq. (2) with the form factors  $F_{1,2}^{L,R}(q^2)$  replaced by  $\bar{F}_{1,2}^{L,R}(q^2)$ . To lowest order in

$\alpha_s$  the penguin amplitude for the decay process

$b \rightarrow q_k g \rightarrow q_k q' \bar{q}'$  ( $q_k q_i \bar{q}_j$ ) $_{i=j}$  is

$$M^{P_{eng}} = -i(\alpha_s / \pi) [\bar{u}_k(p_k) T^a \Lambda_{\mu} u_b(p_b)] \times [\bar{u}_{q'}(p_{q'}) \gamma^{\mu} T^a v_{\bar{q}'}(p_{\bar{q}'}), \quad (4)$$

where  $\alpha_s = g_s^2 / 4\pi$  and

$$\Lambda_{\mu} \equiv \gamma_{\mu} [F_1^L(q^2) P_L + F_1^R(q^2) P_R] + (i\sigma_{\mu\nu} q^{\nu} / q^2) \times [F_2^L(q^2) P_L + F_2^R(q^2) P_R], \quad (5)$$

Similarly, for  $\bar{b} \rightarrow \bar{q}_k q' \bar{q}'$ , the amplitude is

$$\bar{M}^{P_{eng}} = i(\alpha_s / \pi) [\bar{v}_k(p_k) T^a \bar{\Lambda}_{\mu} v_b(p_b)] \times [\bar{u}_{q'}(p_{q'}) \gamma_{\mu} T^a v_{\bar{q}'}(p_{\bar{q}'}), \quad (6)$$

where  $\bar{\Lambda}_{\mu}$  is obtained from eq. (5) by the replacement of all the  $F(q^2)$  form factors by  $\bar{F}(q^2)$  form factors.

The top quark dominates in the sum for  $F_2$ , hence at value of  $q^2$  (a good approximation), we have  $F_2^L(q^2) \approx F_2^L(0)$  and  $F_2^R(q^2) \approx F_2^R(0)$  [3], so

$$F_1^L(q^2) = (G_F / \sqrt{2}) \sum_{i=u,c,t} V_{ik}^* V_{ib} f_1(x_i, q^2), \quad F_1^R(0) = 0, \quad (7)$$

$$F_2^L(0) / m_q = F_2^R(0) / m_b = (G_F / \sqrt{2}) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i), \quad (8)$$

where  $x_i \equiv m_i^2 / M_W^2$  ( $i = u, c, t$ ) and

$$f_2(x) = -(x/4(1-x)^4)[2+3x-6x^2+x^3+6x \ln x], \quad (9)$$

$$f_1(x) = (1/12(1-x)^4) \quad (10)$$

$$\times [18x - 29x^2 + 10x^3 + x^4 - (8 - 32x + 18x^2) \ln x],$$

$$f_1(x_i, q^2) = (10/9) - (2/3) \ln x_i + (2/3z_i) - (2(2z_i + 1)/3z_i)g(z_i). \quad (11)$$

Here  $z_i \equiv q^2 / 4m_i^2$  and

$$g(z) = \begin{cases} \sqrt{\frac{1-z}{z}} \arctan\left(\sqrt{\frac{z}{1-z}}\right), & z < 1 \\ \frac{1}{2} \sqrt{\frac{z-1}{z}} [\ln(\frac{\sqrt{z} + \sqrt{z-1}}{\sqrt{z} - \sqrt{z-1}}) - i\pi], & z > 1 \end{cases} \quad (12)$$

For the u quark,  $z_i$  is large and we use the asymptotic form of eq. (11),

$$f_1(x_u, q^2) = (10/9) - (2/3)[\ln(q^2 / M_W^2) - i\pi]. \quad (13)$$

We find  $F_1^L \gg F_1^R$  and  $F_2^R \gg F_2^L$ . For the  $b \rightarrow dq\bar{q}'$  amplitude we find that  $F_1^L$  is dominant. Processes like  $b \rightarrow ds\bar{s}$  and  $\bar{b} \rightarrow \bar{d}s\bar{s}$  are expected to be penguin dominated [4] and  $F_1^L$  dominates all the other form factors. In the  $b \rightarrow sq\bar{q}'$  transition, we again find that  $F_1^L \gg F_1^R$ ,  $F_2^R \gg F_2^L$  and the  $F_1^L$  amplitude to be dominant.

Now, a very important issue is the generation of QCD corrections to penguin operators. Consider, for example, the local operator  $(\bar{u}_\alpha b_\alpha)_{V-A} (\bar{d}_\beta u_\beta)_{V-A}$ , which is directly induced by W-boson exchange. In this case, additional QCD correction diagrams, with a gluon contribute and as a consequence four operators, instead of two, are involved in the mixing under renormalization. These are [5, 6]:

$$\begin{aligned} Q_3 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}, \\ Q_4 &= (\bar{d}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V-A}, \\ Q_5 &= (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A}, \\ Q_6 &= (\bar{d}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V+A}, \end{aligned} \quad (14)$$

$\alpha$  and  $\beta$  are colour indices. The sum over q runs over all quark flavours that exist in the effective theory in

question. Since the gluon coupling is of course flavor conserving, it is clear that penguins cannot be generated from the operator current due to the gluon coupling in the lower part. For convenience, this vector structure is decomposed into a (V-A) and a (V+A) part according to chiral representation,

$$(\bar{q}_{i\alpha} b_\alpha)_{V\mp A} (\bar{q}_{k\beta} q_{j\beta})_{V\mp A} = (\bar{q}_{i\alpha} \gamma^\mu ((1 \mp \gamma_5) / 2) b_\alpha) (\bar{q}_{k\beta} \gamma_\mu ((1 \mp \gamma_5) / 2) q_{j\beta}) \quad (15)$$

and in the terms of two component spinors are given by,

$$\begin{aligned} \bar{q}_{i\alpha} \gamma^\mu ((1 - \gamma_5) / 2) b_\alpha &= q_{i\alpha L}^\dagger \tilde{\sigma}^\mu b_{\alpha L}, \\ \bar{q}_{k\beta} \gamma_\mu ((1 - \gamma_5) / 2) q_{j\beta} &= q_{k\beta L}^\dagger \tilde{\sigma}_\mu q_{j\beta L}, \\ \bar{q}_{i\alpha} \gamma^\mu ((1 + \gamma_5) / 2) b_\alpha &= q_{i\alpha R}^\dagger \sigma^\mu b_{\alpha R}, \\ \bar{q}_{k\beta} \gamma_\mu ((1 + \gamma_5) / 2) q_{j\beta} &= q_{k\beta R}^\dagger \sigma_\mu q_{j\beta R}. \end{aligned} \quad (16)$$

For each of these, two different colour forms arise due to the colour structure of the exchanged gluon. The amplitude (4) can be written [7],

$$\begin{aligned} M^{Peng} &= -i(G_F / \sqrt{2}) \{ (\alpha_s (M_W / 8\pi) \\ &\times [ \sum_{i=u,c} V_{ib} V_{iq}^* f_1(x_i, q^2) + V_{ib} V_{iq}^* f_1(x_t) ] Q_P \\ &+ (1/2) \sum_{i=u,c,t} V_{ib} V_{iq}^* f_2(x_i) Q_8 \}, \end{aligned} \quad (17)$$

$Q_8$  is the chromomagnetic dipole operator:

$$\begin{aligned} Q_8 &= 4\alpha_s^2 m_b [\bar{q}_{i\alpha} \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta}^a b_\beta] \\ &\times (q_\nu / q^2) [\bar{q}_{j\beta} \gamma_\mu T^a q_k], \end{aligned} \quad (18)$$

Here

$$Q_P = Q_4 + Q_6 - (1/3)(Q_3 + Q_5). \quad (19)$$

As a weak decay under the presence of the strong interaction B meson decays require special techniques [8]. The main tool to calculate such B meson decays is the effective Hamiltonian theory [9,10]. It is a two - step program, starting with an operator product expansion (OPE) and performing a renormalization group equation (RGE) analysis afterwards [10,11,12]. The necessary machinery has been developed over the last years.

The derivation starts as follows: If the kinematics of the decay are of the kind that the masses of the internal particle  $M_i$  are much larger than the external momenta  $P$ ,  $M_i^2 \gg p^2$ , then the heavy particle can be integrated out. This concept takes concrete form with the functional integral formalism. It means that the heavy particles are removed as dynamical degrees of freedom from the theory hence their fields do not appear in the (effective) Lagrangian anymore. Their residual effect lies in the generated effective vertices [13]. In this way, an effective low energy theory can be constructed from a full theory like the Standard Model [14]. A well known example is the four-Fermi interaction, where the W-boson propagator is made local for  $M_W^2 \gg q^2$  ( $q$  denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2 - M_W^2) \rightarrow ig_{\mu\nu}[(1/M_W^2) + (q^2/M_W^4) + \dots], \quad (20)$$

where the ellipses denote terms of higher order in  $1/M_W$ .

Apart from the t quark, the basic framework for weak decays quarks is the effective field theory relevant for scales  $M_W, M_Z, M_t \gg \mu$  [9,15]. This framework, as stated before, brings in local operators, which govern "effectively" the transition in question. From decaying quark point of view, it represents the generalization of the Fermi theory as formulated by Sudershan and Marshak and Feynman and Gell-Mann forty years ago.

It is well known that the decay amplitude is the product of two different parts, whose phases are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. Indeed the simplest effective Hamiltonian without QCD effects ( $b \rightarrow ud\bar{u}$ ) is

$$H_{eff}^0 = 2\sqrt{2}G_F V_{ub}V_{ud}^* Q_1, \quad (21)$$

where  $G_F$  is the Fermi constant,  $V_{ij}$  are the relevant CKM factors and

$$Q_1 = (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{d}_\beta u_\beta)_{V-A}, \quad (22)$$

is a  $(V-A)$ ,  $(V-A)$  is current-current local operator. This simple tree amplitude introduces a new operator  $Q_2$  and is modified by the QCD effect to

$$H_{eff} = 2\sqrt{2}G_F V_{ub}V_{ud}^* (C_1 Q_1 + C_2 Q_2), \quad (23)$$

Here

$$Q_2 = (\bar{u}_\beta b_\alpha)_{V-A} (\bar{d}_\alpha u_\beta)_{V-A}, \quad (24)$$

where  $C_1$  and  $C_2$  are the Wilson coefficients. The situation in the Standard Model is, however, more complicated because of the presence of additional interactions in particular penguins which effectively generate new operators. These are in particular the gluon, photon and  $Z^0$ -boson exchanges and penguin b quark contributions.

Consequently, the relevant effective Hamiltonian for B-meson decays generally involves several operators  $Q_i$  with various colour and Dirac structures which are different from  $Q_1$ . The operators can be grouped into three categories [16]:  $i = 1, 2$  – current-current operators,  $i = 3, \dots, 6, 8$  – gluonic penguin operators and  $i = 7', \dots, 10'$  – Electroweak penguin operators. Moreover, each operator is multiplied by a calculable Wilson coefficient  $C_i(\mu)$ :

$$H_{eff} = 2\sqrt{2}G_F \left[ \sum_{i=1}^{6,8} d_i(\mu) Q_i(\mu) + \sum_{i=7}^{10} d'_i(\mu) Q'_i(\mu) \right], \quad (25)$$

where the scale  $\mu$  is discussed below,  $d_i(\mu) = V_{CKM} C_i(\mu)$  and  $V_{CKM}$  denotes the relevant

CKM factors that are:

$$\begin{aligned} d_{1,2} &= V_{ib} V_{jk}^* C_{1,2}, \\ d_{3,\dots,6} &= -V_{tb} V_{tk}^* C_{3,\dots,6}, \\ d_8 &= -(\alpha_s / 4\pi) V_{tb} V_{tk}^* C_8, \\ d'_{7,\dots,10} &= -(3/2) e_q V_{tb} V_{tk}^* C_{7,\dots,10}. \end{aligned} \quad (26)$$

For tree-level the  $d_{1,2}$  coefficients are:

$$d_1 = V_{ib} V_{kj}^*, \quad d_2 = 0. \quad (27)$$

and for the effective penguin model the  $d_{3,\dots,6,8}$  coefficients are:

$$\begin{aligned} d_3 = d_5 &= (1/6)(\alpha_s / 4\pi) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_1(x_i), \\ d_4 = d_6 &= -3d_3, \\ d_8 &= -(m_b / 2)(\alpha_s / 4\pi) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i). \end{aligned} \quad (28)$$

At this stage it should be mentioned that the usual Feynman diagram containing full  $W$  propagators,  $Z^0$  propagators and top quark propagators really represent the happening at scales  $O(M_W)$  whereas the true picture of a decaying hadron is more correctly described by the local operators in question. Thus, whereas at scale  $O(M_W)$ , we have to deal with the full six-quark theory containing the photon, weak gauge bosons and gluon, at scale  $O(1\text{GeV})$  the relevant effective theory contains only three light quarks  $u, d$ , and  $s$ , gluons and the photon. At intermediate energy scales  $\mu = O(m_b)$  and  $\mu = O(m_c)$  relevant for beauty and charm decays, effective five-quark and effective four-quark theories have to be considered, respectively [17].

The usual procedure then is to start at a high energy scale  $O(M_W)$  and consecutively integrate out the heavy degrees of freedom (heavy with respect to the relevant scale  $\mu$ ) from explicitly appearing in the theory. The word explicitly is very essential here. The heavy fields did not disappear. Their effects are merely hidden in the effective gauge coupling constants, running masses and most importantly the coefficients describing the effective strength of the operators at a scale  $\mu$ , the Wilson coefficient functions  $C_i(\mu)$  [5,9,10,18]. It is straightforward to apply  $H_{eff}$  to B- and D-meson decays as well by changing the quark flavours appropriately.  $\mu$  is some low-energy scale of  $O(1\text{GeV})$ ,  $O(m_c)$  and  $O(m_b)$  for K, D, and B meson decays, respectively. The argument  $\mu$  of the operators  $Q_i(\mu)$  means that their matrix elements are to be normalized at scale  $\mu$ .

## 2. Magnetic dipole amplitude of $b \rightarrow q_i q_k \bar{q}_j$

A charge particle in orbital motion generates a magnetic

dipole moment of a magnitude proportional to its orbital angular momentum. Further more, a particle with intrinsic angular momentum or spin has an intrinsic magnetic moment. The magnetic dipole term in the penguin amplitude, according to eq. (5), is

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu}q^\nu / q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (29)$$

Also, according to eq. (8) magnetic (dipole) form factor at  $q^2 = 0$  ( $q^2 / M_W^2 \ll 1$ ) is

$$\frac{F_2^L(0)}{m_k} = \frac{F_2^R(0)}{m_b} = \frac{g_s^2}{8M_W^2} \sum_i V_{ik}^* V_{ib} f_2(x_i). \quad (30)$$

The top quark is dominant for  $F_2^R(0)$ , so we can write

$$F_2^R(0) = m_b (G_F / \sqrt{2})(V_{ik}^* V_{ib}) f_2(x_t). \quad (31)$$

Here  $f_2(x_t)$  defined by (9) and  $x_t = m_t^2 / M_W^2$ , also we saw that  $F_2^L(0) \ll F_2^R(0)$ , because  $m_k \ll m_b$  so the magnetic dipole term becomes:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu}q^\nu / q^2)F_2^R(0)P_R. \quad (32)$$

Putting in the penguin amplitude, according to eq. (4),

$$M^{dip} = \frac{g_s^2}{4\pi^2} \times [\bar{u}_k(p_k)T^a (i\sigma_{\mu\nu}q^\nu / q^2)F_2^R(0)P_R u_b(p_b)] \times [\bar{u}_i(p_i)\gamma^\mu T^a v_j(p_j)]. \quad (33)$$

The magnetic dipole of penguin amplitude is given by (see App.A),

$$M^{dip} = A_8 d_8 [\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle + \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle], \quad (34)$$

here

$$d_8 = -(2\sqrt{2}G_F)(m_b / 2)(\alpha_s / 4\pi) \sum_i V_{ik}^* V_{ib} f_2(x_i),$$

$$A_8 = (1/\sqrt{2})(4/3)(m_b / q^2). \quad (35)$$

Now we must calculate each terms of the above equation for b spins project -1/2 and 1/2, then squaring these terms and summing up all of them and finally, averaging. The penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by (see App.A),

$$M_{(+1/2)}^{dip} = A_8 d_8 \{ [-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2) + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] + [-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2) + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \}, \quad (36)$$

$$M_{(-1/2)}^{dip} = A_8 d_8 \{ [-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2) - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] + [-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2) - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \}. \quad (37)$$

### 3. Effective Hamiltonian decay rates of $b \rightarrow q_i q_k \bar{q}_j$

The effective  $\Delta B = 1$  Hamiltonian at scale  $\mu = O(m_b)$  for tree plus penguin and including the electroweak penguin and the magnetic dipole term is [5,6,9],

$$H_{eff}^{\Delta B=1} = 2\sqrt{2}G_F \{ [d_{1c}(\mu)Q_1^c(\mu) + d_{2c}(\mu)Q_2^c(\mu)] + [d_{1u}(\mu)Q_1^u(\mu) + d_{2u}(\mu)Q_2^u(\mu)] - [\sum_{i=3}^{6,8} d_i(\mu)Q_i(\mu) + \sum_{i=7}^{10} d'_i(\mu)Q'_i(\mu)] \}. \quad (38)$$

Here  $d_1, \dots, d_6, d'_7, \dots, d'_{10}$  are defined by (26),  $d_{1,2c,u} = d_{1,2}(i = j = c, u)$  and index  $k$  refer to d or s. The decay rate is given by (see App.B)

$$d^2\Gamma_{Q_1, \dots, Q_6} / dx dy = \Gamma_{0b} I_{ps}^{EH}, \quad (39)$$

here

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3, \quad (40)$$

where

$$I_{ps}^1 = 6xy \cdot f_{ab} \cdot (1 - h_{abc}), \\ I_{ps}^2 = 6xy \cdot f_{bc} \cdot (1 + h_{bca}), \\ I_{ps}^3 = 6xy \cdot f_{ac} \cdot h_{xa} \cdot h_{yc}. \quad (41)$$

and

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2, \\ \alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2, \\ \alpha_3 = \text{Re}\{ (3d_1 + d_2 + d_3 + 3d_4) d_6^* + (d_1 + 3d_2 + 3d_3 + d_4) d_5^* \}. \quad (42)$$

Here  $\Gamma_{0b}, f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{xa}$  and  $h_{yc}$  defined in Appendix B.

#### 3. 1. Tree-Level

first of all, we obtain the decay rates of tree-level, without QCD corrections, from eq. (39) by choosing

$$d_1 = V_{ib} V_{jk}^* \quad \text{and} \quad d_2 = d_3 = \dots = d_6 = 0, \quad \text{so}$$

$\alpha_1 = 3d_1^2, \alpha_2 = 0$  and  $\alpha_3 = 0$ .  $Q_1$  is the conventional four-Fermi interaction operator thus eq. (39) reduces to  $\Gamma_{Q_1, \dots, Q_6} = \Gamma_{0b} I_{ps}^{EH}$  ( $I_{ps}^{EH} = \alpha_1 I_{ps}^1$ ), so

$$\Gamma_{tree} = 3\Gamma_{0b} |V_{ib} V_{jk}^*|^2 I_{ps}^1. \quad (43)$$

#### 3. 2. Pure Penguin

one can obtain the decay rates of pure penguin, by selecting  $d_1 = d_2 = 0$ , so that  $\alpha_1$  and  $\alpha_3$  reduce to,

$$\alpha_1 = |d_3 + d_4|^2 + 2|d_3|^2 + 2|d_4|^2, \\ \alpha_3 = \text{Re}\{ (d_3 + 3d_4) d_6^* + (3d_3 + d_4) d_5^* \}. \quad (44)$$

$\alpha_2$  and the decay rate keep the form eq. (42).

#### 4. Effective Hamiltonian of electroweak penguin decay rates

The generalization of the  $\Delta B=1$  Hamiltonian in pure QCD to incorporate electroweak penguin operators is the sum of the  $Q_1, \dots, Q_6, Q_7', \dots, Q_{10}'$  eq. (38). The  $\Delta B=1$  Wilson coefficients for  $Q_1^{u,c}, Q_2^{u,c}, Q_3, \dots, Q_6, Q_7', \dots, Q_{10}'$  in the mixed case of QCD and QED. Therefore the discussion of  $C_1, \dots, C_6$  is also valid for the present case. It was observed that, all of the terms  $Q_1, Q_2, Q_3, Q_4$  had a form Left-Left handed. Terms  $Q_1, Q_2$  have a form  $\langle i|_L \tilde{\sigma}^\mu |b\rangle_L \langle k|_L \tilde{\sigma}_\mu |j\rangle_L$ , and terms  $Q_3, Q_4$  have a form  $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_L \tilde{\sigma}_\mu |j\rangle_L$ . Also terms  $Q_5, Q_6$  have a form Left-Right handed,  $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_R \sigma_\mu |j\rangle_R$ . We consider that terms  $Q_7', Q_8'$  have a form Left-Right handed and terms  $Q_9', Q_{10}'$  have a form Left-Left handed. Therefore terms  $Q_7', Q_8'$  have a form  $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_L \tilde{\sigma}_\mu |j\rangle_L$ , and terms  $Q_9', Q_{10}'$  have a form  $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_R \sigma_\mu |j\rangle_R$ . Thus the partial decay rate including electroweak penguin is the same as eq. (39) with different constants  $\alpha_1, \alpha_2$  and  $\alpha_3$ ,

$$\begin{aligned} \alpha_1 &= |d_1 + d_2 + d_3 + d_4 + d_9' + d_{10}'|^2 \\ &\quad + 2|d_1 + d_4 + d_{10}'|^2 + 2|d_2 + d_3 + d_9'|^2, \\ \alpha_2 &= |d_5 + d_6 + d_7' + d_8'|^2 + 2|d_5 + d_8'|^2 + 2|d_6 + d_7'|^2, \\ \alpha_3 &= \text{Re}\{(3d_1 + d_2 + d_3 + 3d_4 + d_9' + 3d_{10}') (d_5^* + d_8'^*)\}, \end{aligned} \quad (45)$$

where  $d_1, \dots, d_6, d_7', \dots, d_{10}'$  defined by eq. (26) and  $e_q$  is the quark electric charge which is given by

$$e_{u,c,t} = 2/3, \quad e_{d,s,b} = -(1/3). \quad (46)$$

#### 5. Effective Hamiltonian of magnetic dipole decay rates

Now the aim is to calculate the decay rates of  $b \rightarrow q_i q_k \bar{q}_j$  according to the effective Hamiltonian ( $Q_1, \dots, Q_6$ ), including magnetic dipole ( $Q_8$ ) terms. We obtained the amplitude of operators  $Q_1, \dots, Q_6$ , and the amplitude of magnetic dipole ( $Q_8$ ) terms. Adding the amplitudes, we calculated the decay rates of operators of the effective Hamiltonian including magnetic dipole terms ( $Q_1, \dots, Q_6, Q_8$ ). The amplitude of the effective Hamiltonian including magnetic dipole is given by (see App.C)

$$\begin{aligned} |M^{tot}|_{spin-ave}^2 &= \frac{1}{4} [g_1 - 2v_k v_i (g_4 - g_5) \cos(\theta_k - \theta_i) \\ &\quad + g_2 + 2v_j v_k (g_6 + g_7) \cos(\theta_j - \theta_k) \\ &\quad - 2\sqrt{1-v_i^2} \sqrt{1-v_j^2} g_3 \\ &\quad - 2v_j v_i (g_8 - g_9) \cos(\theta_j - \theta_i)], \end{aligned} \quad (47)$$

In order to check the above equation, one can obtain the amplitude of tree-level and the effective Hamiltonian ( $Q_1, \dots, Q_6$ ). The amplitude of tree-level ( $d_2 = d_3 = d_4 = d_5 = d_6 = d_8 = 0$ ) is given by

$$\begin{aligned} |M^{tot}|_{spin-ave, TL}^2 &= \frac{1}{4} [2h_1^2 - 2h_1^2 v_i v_k \cos(\theta_k - \theta_i) + 0 + 0] \\ &= 3d_1^2 \cdot \frac{1}{2} (1 - v_i v_k \cos(\theta_k - \theta_i)), \end{aligned} \quad (48)$$

and the amplitude of the effective Hamiltonian ( $d_8 = 0$ ) is given by

$$\begin{aligned} |M^{tot}|_{spin-ave, EH}^2 &= (h_1^2 / 2) [1 - v_k v_i \cos(\theta_k - \theta_i)] \\ &\quad + (h_3^2 / 2) [1 + v_j v_k \cos(\theta_j - \theta_k)] \\ &\quad - (\sqrt{1-v_i^2} \sqrt{1-v_j^2} / 4) (2h_1 h_3). \end{aligned} \quad (49)$$

The differential of decay rate of the effective Hamiltonian plus magnetic dipole eq. (47) is given by

$$\frac{d^2 \Gamma}{dx dy} = \frac{G_F^2 M_b^2}{192 \pi^3} \frac{1}{2} (I_1 + I_2 + I_3), \quad (50)$$

where

$$\begin{aligned} I_1 &= 6xy f_{ab} [g_1 - 2(g_4 - g_5) h_{abc}], \\ I_2 &= 6xy f_{bc} [g_2 + 2(g_6 + g_7) h_{bca}], \\ I_3 &= 6xy f_{ac} [-2g_3 h_{xa} h_{yc} - 2(g_8 - g_9) h_{acb}] \end{aligned} \quad (51)$$

and  $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}, h_{yc}$  defined by (B-16). Now the decay rates of eq. (50) is checked for Tree-Level and effective Hamiltonian ( $Q_1, \dots, Q_6$ ). Putting  $d_2 = d_3 = d_4 = d_5 = d_6 = d_8 = 0$  in eq. (50) the decay rate of Tree-Level is obtained which is the same as eq. (43) and putting  $d_8 = 0$  in eq. (33) the decay rate of the effective Hamiltonian is found to be the same as eq. (39).

#### 6. Effective Hamiltonian of electroweak penguin and magnetic dipole decay rates

The partial decay rate of the effective Hamiltonian plus Magnetic Dipole was given by eq. (50). Now the partial decay rate including electroweak penguin is obtained. In this case, the electroweak penguin operators ( $d_7', d_8', d_9', d_{10}'$ ) should be included in the as did before for obtaining the partial decay rate is given by

$$\frac{d^2 \Gamma}{dx dy} = \frac{G_F^2 M_b^2}{192 \pi^3} \frac{1}{2} (I_1 + I_2 + I_3), \quad (52)$$

where  $I_1, I_2, I_3$  defined by eq. (51) and  $h_1, h_2, h_3$  defined by,

**Table 1.** Effective Wilson coefficients  $C_i^{eff}$  at the renormalization scale  $\mu = 2.5$  for the various  $b \rightarrow q$  ( $\bar{b} \rightarrow \bar{q}$ ) transitions.

|             | $b \rightarrow d$ | $\bar{b} \rightarrow \bar{d}$ | $b \rightarrow s$ | $\bar{b} \rightarrow \bar{s}$ |
|-------------|-------------------|-------------------------------|-------------------|-------------------------------|
| $C_1^{eff}$ | 1.1679+0.0000i    | 1.1679+0.0000i                | 1.1679+0.0000i    | 1.1679+0.0000i                |
| $C_2^{eff}$ | -0.3525+0.0000i   | -0.3525+0.0000i               | -0.3525+0.0000i   | -0.3525+0.0000i               |
| $C_3^{eff}$ | 0.0217+0.0018i    | 0.0234+0.0047i                | 0.0232+0.0030i    | 0.0231+0.0029i                |
| $C_4^{eff}$ | -0.0498-0.0054i   | -0.0543-0.0142i               | -0.0535-0.0091i   | -0.0531-0.0086i               |
| $C_5^{eff}$ | 0.0156+0.0018i    | 0.0173+0.0047i                | 0.0171+0.0030i    | 0.0170+0.0029i                |
| $C_6^{eff}$ | -0.0625-0.0054i   | -0.0678-0.0142i               | -0.0670-0.0091i   | -0.0667-0.0086i               |

**Table 2.** Branching ratios (BR) of tree-level  $b \rightarrow q_i q_k \bar{q}_j$  (43) ( $\Gamma_{tot} = 3.0457 \times 10^{-13} \text{ GeV}$ ).

| Process                                | $BR \times 10^{-2}$ | Process                                | $BR \times 10^{-2}$ |
|--|---------------------|--|---------------------|
| $b \rightarrow ce^- \bar{\nu}_e$       | 14.62               | $b \rightarrow ue^- \bar{\nu}_e$       | 0.231               |
| $b \rightarrow c\mu^- \bar{\nu}_\mu$   | 14.62               | $b \rightarrow u\mu^- \bar{\nu}_\mu$   | 0.231               |
| $b \rightarrow c\tau^- \bar{\nu}_\tau$ | 0.714               | $b \rightarrow u\tau^- \bar{\nu}_\tau$ | 0.084               |
| $b \rightarrow cd\bar{u}$              | 49.02               | $b \rightarrow ud\bar{u}$              | 0.725               |
| $b \rightarrow cs\bar{c}$              | 16.13               | $b \rightarrow uc\bar{c}$              | 0.019               |
| $b \rightarrow cd\bar{c}$              | 0.857               | $b \rightarrow us\bar{u}$              | 0.531               |
| $b \rightarrow cs\bar{u}$              | 2.352               | $b \rightarrow us\bar{c}$              | 0.355               |

$$\begin{aligned}
h_1 &= \sqrt{|d_1 + d_2 + d_3 + d_4 + d'_9 + d'_{10}|^2 + 2|d_1 + d_4 + d'_{10}|^2 + 2|d_2 + d_3 + d'_9|^2} \\
h_2 &= A_8 d_8, \\
h_3 &= \sqrt{|d_5 + d_6 + d'_7 + d'_8|^2 + 2|d_5 + d'_8|^2 + 2|d_6 + d'_7|^2}, \\
\end{aligned} \tag{53}$$

## 7. Numerical results

As an example of the use of the above formalism, we use the standard Particle Data Group [19] parameterization of the CKM matrix with the central values

$$\theta_{12} = 0.221, \theta_{13} = 0.0035, \theta_{23} = 0.041,$$

and choose the CKM phase  $\delta_{13}$  to be  $\pi/2$ . Following Ali and Greub [16], we treat internal quark masses in tree-level loops with the values (GeV)  $m_b = 4.88$ ,  $m_s = 0.2$ ,  $m_d = 0.01$ ,  $m_u = 0.005$ ,  $m_c = 1.5$ ,  $m_e = 0.0005$ ,  $m_\mu = 0.1$ ,  $m_\tau = 1.777$  and  $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$ . The

effective Wilson coefficients  $C_i^{eff}$  at the renormalization scale  $\mu = 2.5 \text{ GeV}$  for the various  $b \rightarrow q$  ( $\bar{b} \rightarrow \bar{q}$ ) transitions, shown in the Table 1 [7]. Also, following H.Y.Cheng [20], [21,5,10] and [22, 23, 5, 10], we choose

the effective Wilson coefficients of  $C_7^{eff} - C_{10}^{eff}$ ,

$$C_7^{eff} = -(0.0276 + i0.0369)\alpha_e, \quad C_8^{eff} = 0.054\alpha_e,$$

$$C_9^{eff} = -(1.138 + i0.0369)\alpha_e, \quad C_{10}^{eff} = 0.263\alpha_e,$$

here  $\alpha_e = 1/137$ , that is the electromagnetic coupling constant. In order to obtain the total rates at the tree-level a sum is made over the b-quark decay rates. The total decay rate and branching ratios of several of semileptonic and hadronic modes are given in Table 2. We see that the modes  $b \rightarrow cl\nu$ ,  $b \rightarrow cd\bar{u}$  and  $b \rightarrow cs\bar{c}$  are dominant. The total b-quark decay rate at the tree-level is given by

$$\Gamma_{total}^T = \Gamma_{Semileptonic}^T + \Gamma_{Hadronic}^T = 3.0457 \times 10^{-13} \text{ GeV}.$$

It is observed that the decay rate for the antiparticle  $\bar{b} \rightarrow \bar{u}\bar{d}\bar{u}$  is greater than the particle decay rate  $b \rightarrow ud\bar{u}$ , and the antiparticle decay rate  $\bar{b} \rightarrow \bar{c}\bar{d}\bar{c}$  is less than the particle decay rate  $b \rightarrow cd\bar{c}$ , and so on. We consider that the modes  $b \rightarrow c\bar{u}\bar{d}$ , and  $b \rightarrow c\bar{c}\bar{s}$  are dominant.

The branching ratios of the effective Hamiltonian ( $Q_1, \dots, Q_6$ ) for particles and antiparticles are collected in Table (3). In this case modes  $b \rightarrow cd\bar{u}$  and  $b \rightarrow cs\bar{c}$  are dominant. Also, the branching ratios including the

**Table 3.** Branching ratios ( $BR \times 10^{-2}$ ) of tree-level ( $T$ ) eq. (43), Effective Hamiltonian ( $EH$ ) eq. (39), Effective Hamiltonian including Electroweak Penguin ( $EH + EP$ ) eq. (45), Effective Hamiltonian including Magnetic Dipole ( $EH + MD$ ) eq. (50) and Effective Hamiltonian including Electroweak Penguin and Magnetic Dipole ( $EH + EP + MD$ ) eq. (52) of the particles and antiparticles for the various  $b \rightarrow q$  ( $\bar{b} \rightarrow \bar{q}$ ) transitions. The total decay rates are in unit of  $10^{-13} \text{ GeV}$ .

| $\Gamma_{tot} \rightarrow$            | 3.0457  | 3.404       | 3.497         | 3.529         | 3.637              |
|---------------------------------------|---------|-------------|---------------|---------------|--------------------|
| Process                               | ( $T$ ) | ( $E - H$ ) | ( $EH + EP$ ) | ( $EH + MD$ ) | ( $EH + EP + MD$ ) |
| $b \rightarrow ud\bar{u}$             | 0.725   | 0.899       | 0.872         | 0.903         | 0.914              |
| $b \rightarrow cd\bar{c}$             | 0.857   | 0.995       | 0.956         | 0.992         | 1.102              |
| $b \rightarrow cd\bar{u}$             | 49.02   | 49.83       | 48.51         | 50.35         | 52.26              |
| $b \rightarrow ud\bar{c}$             | 0.019   | 0.024       | 0.024         | 0.019         | 0.038              |
| $b \rightarrow us\bar{u}$             | 0.531   | 0.603       | 0.544         | 0.680         | 0.714              |
| $b \rightarrow cs\bar{c}$             | 16.13   | 17.09       | 18.19         | 17.780        | 18.69              |
| $b \rightarrow us\bar{c}$             | 0.355   | 0.658       | 0.578         | 0.697         | 0.746              |
| $b \rightarrow cs\bar{u}$             | 2.352   | 2.623       | 3.716         | 0.521         | 3.962              |
| $\bar{b} \rightarrow \bar{u}d\bar{u}$ | 0.725   | 0.943       | 0.915         | 0.951         | 0.988              |
| $\bar{b} \rightarrow \bar{c}d\bar{c}$ | 0.857   | 0.987       | 0.958         | 1.124         | 1.658              |
| $\bar{b} \rightarrow \bar{c}d\bar{u}$ | 49.02   | 49.80       | 48.49         | 50.31         | 50.87              |
| $\bar{b} \rightarrow \bar{u}d\bar{c}$ | 0.019   | 0.023       | 0.022         | 0.032         | 0.045              |
| $\bar{b} \rightarrow \bar{u}s\bar{u}$ | 0.531   | 0.595       | 0.539         | 0.675         | 0.721              |
| $\bar{b} \rightarrow \bar{c}s\bar{c}$ | 16.13   | 17.10       | 18.18         | 17.791        | 18.38              |
| $\bar{b} \rightarrow \bar{u}s\bar{c}$ | 0.355   | 0.655       | 0.577         | 0.694         | 0.762              |
| $\bar{b} \rightarrow \bar{c}s\bar{u}$ | 2.352   | 2.623       | 3.707         | 2.512         | 4.130              |

electroweak penguin ( $Q_1, \dots, Q_6, Q'_7, \dots, Q'_{10}$ ) are shown as well and those of the pure Penguin are given in Table 3 and 4 respectively. It is seen that, in the pure Penguin decays, modes  $b \rightarrow ss\bar{s}$  and  $b \rightarrow sd\bar{d}$  are dominant. Also, it is observed that, terms of Current-Current plus Penguin operators dominate as compared with the electroweak Penguin operators.

The branching ratios of the effective Hamiltonian plus magnetic dipole, and the effective Hamiltonian plus electroweak Penguin plus magnetic dipole are shown in Table 3 as well. It shows that the electroweak Penguin plus magnetic dipole term is small and can be neglected in the total decay rate. The total decay rate of the effective Hamiltonian ( $Q_1, \dots, Q_6$ ) of particle and antiparticle are given by

$$\Gamma_{total(Q_1, \dots, Q_6)}^{EH} = 3.404 \times 10^{-13} \text{ GeV}.$$

In addition, the total decay rate of particles and antiparticles including electroweak penguin ( $Q_1, \dots, Q_6, Q'_7, \dots, Q'_{10}$ ) for b-quark decays are given by

$$\Gamma_{total(Q_1, \dots, Q'_{10})}^{EH+EP} = 3.497 \times 10^{-13} \text{ GeV}.$$

Also the total decay rate of particles and antiparticles including effective Hamiltonian and magnetic dipole ( $Q_1, \dots, Q_6, Q_8$ ) and the effective Hamiltonian, electroweak penguin and Magnetic Dipole ( $Q_1, \dots, Q_6, Q_8, Q'_7, \dots, Q'_{10}$ ) for b-quark decays are given by

$$\Gamma_{total(Q_1, \dots, Q_8)}^{EH+MD} = 3.526 \times 10^{-13} \text{ GeV},$$

$$\Gamma_{total(Q_1, \dots, Q_8, Q'_{10})}^{EH+EP+MD} = 3.637 \times 10^{-13} \text{ GeV}.$$

The total decay rates of pure penguin mode particles and antiparticles is

$$\Gamma_{total(Q_3, \dots, Q_6)}^P = 2.479 \times 10^{-15} \text{ GeV}.$$

It is seen that for pure penguin modes, the decay rates of particles are less than those of antiparticles (see Table 4). It is interesting if a comparison is made among the decay rate of the Tree-Level ( $T$ ) (see eq. (43)), effective Hamiltonian ( $Q_1, \dots, Q_6$ ) ( $EH$ ) (see eq. (39)), effective Hamiltonian plus electroweak Penguin ( $Q_1, \dots, Q_6, Q'_7, \dots, Q'_{10}$ ) ( $EH + EP$ ) (see (45)), effective Hamiltonian plus magnetic dipole ( $EH + MD$ ) (see eq. (50)) and the effective Hamiltonian plus electroweak penguin plus magnetic dipole ( $Q_1, \dots, Q_6, Q_8, Q'_7, \dots, Q'_{10}$ ) ( $EH + EP + MD$ ) (see eq. (52)) that show at Table 3.

## 8. Conclusions

We obtained the decay rates of the b-quark at the tree-level, penguin, effective Hamiltonian, effective Hamiltonian including electroweak penguin, and for the first time, the effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole terms of the particles and antiparticles for the various

**Table 4.** Branching ratios of pure penguin of effective Hamiltonian of the particles and antiparticles for the various  $b \rightarrow q$  ( $\bar{b} \rightarrow \bar{q}$ ) transitions (44). ( $\Gamma_{tot} = 3.404 \times 10^{-13} \text{ GeV}$ ).

| Process                         | $BR \times 10^{-4}$ | Process                                     | $BR \times 10^{-4}$ |
|---------------------------------|---------------------|---|---------------------|
| $b \rightarrow d\bar{d}\bar{d}$ | 2.408               | $\bar{b} \rightarrow \bar{d}\bar{d}\bar{d}$ | 2.615               |
| $b \rightarrow d\bar{s}\bar{s}$ | 2.687               | $\bar{b} \rightarrow \bar{d}\bar{s}\bar{s}$ | 2.763               |
| $b \rightarrow s\bar{d}\bar{d}$ | 53.872              | $\bar{b} \rightarrow \bar{s}\bar{d}\bar{d}$ | 54.053              |
| $b \rightarrow s\bar{s}\bar{s}$ | 54.517              | $\bar{b} \rightarrow \bar{s}\bar{s}\bar{s}$ | 53.782              |

$b \rightarrow q$  ( $\bar{b} \rightarrow \bar{q}$ ) transitions. According to Table 2, the dominant mode in b-quark in the semileptonic and hadronic decays are,  $b \rightarrow c\ell^- \bar{\nu}_\ell$  ( $\ell \rightarrow e, \mu$ ) and  $b \rightarrow cd\bar{u}$  respectively because the decay rates of  $b \rightarrow c$  channel are very much bigger than those of  $b \rightarrow u$ , since  $V_{cb} \gg V_{ub}$ . In addition, the dominant mode in the pure penguin decays is,  $b \rightarrow s$ . According to Table 4, the branching ratios of pure penguin of the effective Hamiltonian of the particles and antiparticles are close.

The electroweak penguin and magnetic dipole terms are small for b-quark decay rates (electroweak corrections and the magnetic dipole contributions are small) and the decay rate of the tree, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole of the particles and antiparticles are also not very different (see Table 3). The decay rates of  $b^-$  and  $\bar{b}^-$  quark, at the tree-level are exactly the same, but in the pure penguin, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole, are different. For example,  $\Gamma_{b \rightarrow s\bar{d}\bar{d}} < \Gamma_{\bar{b} \rightarrow \bar{s}\bar{d}\bar{d}}$ ,  $\Gamma_{b \rightarrow u\bar{d}\bar{u}} < \Gamma_{\bar{b} \rightarrow \bar{u}\bar{d}\bar{u}}$ ,  $\Gamma_{b \rightarrow c\bar{d}\bar{u}} > \Gamma_{\bar{b} \rightarrow \bar{c}\bar{d}\bar{u}}$  and  $\Gamma_{b \rightarrow c\bar{s}\bar{c}} \approx \Gamma_{\bar{b} \rightarrow \bar{c}\bar{s}\bar{c}}$ , because the total decay rates of  $b^-$  and  $\bar{b}^-$  quark must be equal,  $\Gamma_b^{total} = \Gamma_{\bar{b}}^{total}$ .

Also the decay rates and branching ratios are very similar in all the models but the effective Hamiltonian including electroweak penguin and magnetic dipole total decay rate is about 10% larger than the simple tree or effective Hamiltonian. On the other hand, including the penguin term induced matter antimatter asymmetries. These are largest in the rare decays  $b \rightarrow ud\bar{u}$ , the decay rate of which, is about 7% smaller than that of  $\bar{b} \rightarrow \bar{u}\bar{d}\bar{u}$ . Also the rate  $b \rightarrow su\bar{u}$  is larger than the rate  $\bar{b} \rightarrow \bar{s}\bar{u}\bar{u}$ .

## Appendix A

### Penguin amplitude of magnetic dipole

According to eq. (4) the penguin amplitude is given by

$$M^{dip} = \frac{g_s^2}{4\pi^2} [\bar{u}_k(p_k) T^a (i\sigma_{\mu\nu} q^\nu / q^2) F_2^R(0) P_R u_b(p_b)] \times [\bar{u}_i(p_i) \gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})], \quad (\text{A-1})$$

where

$$\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu] = (i/2)(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad (\text{A-2})$$

and

$$\gamma^\mu \gamma^\nu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0 \end{pmatrix} = \begin{pmatrix} \sigma^\mu \tilde{\sigma}^\nu & 0 \\ 0 & \tilde{\sigma}^\mu \sigma^\nu \end{pmatrix}. \quad (\text{A-3})$$

So

$$\sigma^{\mu\nu} = \frac{i}{2} \begin{pmatrix} \sigma^\mu \tilde{\sigma}^\nu - \sigma^\nu \tilde{\sigma}^\mu & 0 \\ 0 & \tilde{\sigma}^\mu \sigma^\nu - \tilde{\sigma}^\nu \sigma^\mu \end{pmatrix}. \quad (\text{A-5})$$

The wave functions of  $b$  and  $q_k$  are given by

$$\begin{aligned} & \bar{u}_k \sigma_{\mu\nu} [(1 + \gamma_5) / 2] u_b \\ &= \frac{i}{2} \begin{pmatrix} \psi_{kL} \\ \psi_{kR} \end{pmatrix}^\dagger \times \begin{pmatrix} \tilde{\sigma}_\mu \sigma_\nu - \tilde{\sigma}_\nu \sigma_\mu & 0 \\ 0 & \sigma_\mu \tilde{\sigma}_\nu - \sigma_\nu \tilde{\sigma}_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \psi_{bR} \end{pmatrix} \\ &= (i/2) \psi_{kL} (\tilde{\sigma}_\mu \sigma_\nu - \tilde{\sigma}_\nu \sigma_\mu) \psi_{bR}. \end{aligned} \quad (\text{A-6})$$

Putting in the penguin amplitude

$$\begin{aligned} M^{dip} &= -\frac{g_s^2}{8\pi^2} F_2^R(0) (T^a T^a) \\ & \times [\bar{u}_{kL} \frac{q^\nu}{q^2} (\tilde{\sigma}_\mu \sigma_\nu - \tilde{\sigma}_\nu \sigma_\mu) u_{bR}] \\ & \times [\bar{u}_i (\tilde{\sigma}^\mu + \sigma^\mu) v_{\bar{j}}]. \end{aligned} \quad (\text{A-7})$$

Putting  $q^\nu = (p_b - p_k)^\nu$  in the above equation,

$$\begin{aligned} M^{dip} &= \\ & -\frac{g_s^2}{8\pi^2} F_2^R(0) (T^a T^a) \frac{1}{q^2} [(\bar{u}_{kL} \tilde{\sigma}_\mu (\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu) u_{bR}) \\ & - (\bar{u}_{kL} (\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu) \sigma_\mu u_{bR})] \times [\bar{u}_i \tilde{\sigma}^\mu v_{\bar{j}} + \bar{u}_i \sigma^\mu v_{\bar{j}}]. \end{aligned} \quad (\text{A-8})$$

or



$$\begin{aligned}
M^{dip} = & -\frac{g_s^2}{8\pi^2} F_2^R(0)(T^a T^a) \frac{1}{q^2} [\langle k_L | \tilde{\sigma}_\mu (\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu) | b_R \rangle \\
& - \langle k_L | (\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu) \sigma_\mu | b_R \rangle] \\
& \times [\langle i_L | \tilde{\sigma}^\mu | j_L \rangle + \langle i_R | \sigma^\mu | j_R \rangle]. \quad (A-9)
\end{aligned}$$

We know that,

$$\begin{aligned}
\tilde{\sigma}_\mu p_b^\mu | b_L \rangle = m_b | b_R \rangle, \quad \tilde{\sigma}_\mu p_k^\mu | k_L \rangle = m_k | k_R \rangle, \\
\sigma_\nu p_b^\nu | b_R \rangle = m_b | b_L \rangle, \quad \sigma_\nu p_k^\nu | k_R \rangle = m_k | k_L \rangle, \\
\tilde{\sigma}_\mu (\sigma_\nu p_b^\nu) = p_{k\mu}, \quad (\tilde{\sigma}_\mu p_b^\nu) \sigma_\mu = p_{b\mu}. \quad (A-10)
\end{aligned}$$

and

$$(p_b + p_k)_\mu = (2p_b - p_i - p_j)_\mu. \quad (A-11)$$

Also according to conservation of current

$$\begin{aligned}
(p_i + p_j)_\mu [\langle i_L | \tilde{\sigma}^\mu | j_L \rangle + \langle i_R | \sigma^\mu | j_R \rangle] \\
= m_i [\langle i_R | j_L \rangle + \langle i_L | j_R \rangle] \\
- m_j [\langle i_L | j_R \rangle + \langle i_R | j_L \rangle] = 0, \quad (A-12)
\end{aligned}$$

since in the penguin decays is  $m_i = m_j$  and  $|j\rangle$  is the antiparticle, so

$$\tilde{\sigma}_\mu p_{j\mu} | j_L \rangle = -m_j | j_R \rangle. \quad (A-13)$$

Consequently, the magnetic dipole term of penguin amplitude becomes to

$$\begin{aligned}
M^{dip} = & -\frac{g_s^2}{8\pi^2} F_2^R(0)(T^a T^a) \\
& \times \frac{1}{q^2} [m_b \langle k_L | \tilde{\sigma}_\mu | b_L \rangle + m_k \langle k_R | \sigma_\mu | b_R \rangle \\
& - (p_b + p_k)_\mu \langle k_L | b_R \rangle] \times [\langle i_L | \tilde{\sigma}^\mu | j_L \rangle + \langle i_R | \sigma^\mu | j_R \rangle]. \quad (A-14)
\end{aligned}$$

Term  $m_k \langle k_R | \sigma_\mu | b_R \rangle$  was neglected because  $m_k \ll m_b$ , so

$$\begin{aligned}
M^{dip} = & (4/3)d_8(1/q^2)[m_b \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& + m_b \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle \\
& - (p_b + p_k)_\mu \langle k_L | b_R \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& - (p_b + p_k)_\mu \langle k_L | b_R \rangle \langle i_R | \sigma^\mu | j_R \rangle]. \quad (A-15)
\end{aligned}$$

Using (A-12) for the second part of the equation, we can write

$$\begin{aligned}
M^{dip} = & (4/3)d_8(1/q^2)[m_b \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& + m_b \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle \\
& - 2p_{b\mu} \langle k_L | b_R \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& - 2p_{b\mu} \langle k_L | b_R \rangle \langle i_R | \sigma^\mu | j_R \rangle]. \quad (A-16)
\end{aligned}$$

Here  $b$  meson is at the rest ( $p_{b\mu} = (m_b, \vec{0})$ ) and

$$(T^a T^a) = 4/3,$$

$$\begin{aligned}
d_8 = & -\frac{g_s^2}{8\pi^2} F_2^R(0) = -\frac{g_s^2}{8\pi^2} m_b \frac{g_2^2}{8M_W^2} \sum_i V_{ik}^* V_{ib} f_2(x_i), \\
& = -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi) \sum_i V_{ik}^* V_{ib} f_2(x_i). \quad (A-17)
\end{aligned}$$

So, the magnetic dipole of penguin amplitude is given by

$$\begin{aligned}
M^{dip} = & (4/3)d_8(m_b/q^2)[\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& + \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle \\
& - 2\langle k_L | b_R \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle - 2\langle k_L | b_R \rangle \langle i_R | \sigma^\mu | j_R \rangle]. \quad (A-18)
\end{aligned}$$

The  $b$  quark is at the rest and to have spin projection  $-1/2$  along angle  $\theta_b$ , thus the spin projection of  $b$  quark of  $+1/2$  is along  $\theta_b - \pi$  ( $\theta_b \rightarrow \theta_b - \pi$ ),

$$b \text{ spin}(-1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2}) \begin{pmatrix} -\sin(\theta_b/2) \\ \cos(\theta_b/2) \end{pmatrix},$$

$$b \text{ spin}(+1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2}) \begin{pmatrix} \cos(\theta_b/2) \\ \sin(\theta_b/2) \end{pmatrix}.$$

(A-19)

Putting the factor of  $(1/\sqrt{2})$  in the  $M^{dip}$  and neglecting the terms  $\langle k_L | b_R \rangle$ , thus the amplitude of magnetic dipole becomes

$$\begin{aligned}
M^{dip} = & A_8 d_8 [\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\
& + \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle], \quad (A-20)
\end{aligned}$$

here

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2). \quad (A-21)$$

Terms  $(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu)$  and  $(\tilde{\sigma}_\mu)(\sigma^\mu)_{LR}$  for spin  $+1/2$  and  $-1/2$  are obtained by the matrix elements of  $L-L$  handed and  $L-R$  handed for the  $b$  quark

$$\begin{aligned}
\langle -i | \tilde{\sigma}^\mu | b_{(1/2)} \rangle_L \langle -k | \tilde{\sigma}_\mu | -j \rangle_L = \\
\sin((\theta_k - \theta_j - \theta_i)/2) + \sin((\theta_k + \theta_j - \theta_i)/2), \\
\langle -i | \tilde{\sigma}^\mu | b_{(-1/2)} \rangle_L \langle -k | \tilde{\sigma}_\mu | -j \rangle_L = \\
\cos((\theta_k - \theta_j - \theta_i)/2) - \cos((\theta_k + \theta_j - \theta_i)/2), \quad (A-22)
\end{aligned}$$

when dealing with penguin amplitudes, the following matrix elements are needed

$$\begin{aligned}
\langle -i | \tilde{\sigma}^\mu | b_{(1/2)} \rangle_L \langle -k | \sigma_\mu | -j \rangle_R = \\
\sin((\theta_i - \theta_k - \theta_j)/2) - \sin((\theta_i + \theta_k - \theta_j)/2), \\
\langle -i | \tilde{\sigma}^\mu | b_{(-1/2)} \rangle_L \langle -k | \sigma_\mu | -j \rangle_R = \\
\cos((\theta_i - \theta_k - \theta_j)/2) + \cos((\theta_i + \theta_k - \theta_j)/2). \quad (A-23)
\end{aligned}$$

The first term of (A-20) for  $b$  spin project  $-1/2$ , according to Fierz transformation,

$$\langle k_L | \tilde{\sigma}^\mu | b_L \rangle \langle i_L | \tilde{\sigma}_\mu | j_L \rangle = -\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle. \quad (A-24)$$

is given by

$$\begin{aligned} M_{1(-1/2)}^{dip} &\equiv -A_8 d_8 \langle k_L | \tilde{\sigma}^\mu | b_L \rangle \langle i_L | \tilde{\sigma}_\mu | j_L \rangle \\ &= A_8 d_8 [-\cos((\theta_b - \theta_i - \theta_k + \theta_j) / 2) \\ &\quad - \cos((\theta_b - \theta_i + \theta_k - \theta_j) / 2)]. \end{aligned} \quad (\text{A-25})$$

And the first term for b spin project +1/2 is given by

$$\begin{aligned} M_{1(+1/2)}^{dip} &\equiv -A_8 d_8 \langle k_L | \tilde{\sigma}^\mu | b_L \rangle \langle i_L | \tilde{\sigma}_\mu | j_L \rangle \\ &= A_8 d_8 [-\sin((\theta_b + \theta_i - \theta_k - \theta_j) / 2) \\ &\quad + \sin((\theta_b - \theta_i + \theta_k - \theta_j) / 2)]. \end{aligned} \quad (\text{A-26})$$

Also the second term of (A-20) for b spin project -1/2 is given by

$$\begin{aligned} M_{2(-1/2)}^{dip} &\equiv -A_8 d_8 \langle k_L | \tilde{\sigma}^\mu | b_L \rangle \langle i_R | \sigma_\mu | j_R \rangle \\ &= A_8 d_8 [-\cos((\theta_b + \theta_i - \theta_k - \theta_j) / 2) \\ &\quad - \cos((\theta_b - \theta_i + \theta_k - \theta_j) / 2)]. \end{aligned} \quad (\text{A-27})$$

In addition, the second term for b spin project +1/2 is given by

$$\begin{aligned} M_{2(+1/2)}^{dip} &\equiv -A_8 d_8 \langle k_L | \tilde{\sigma}^\mu | b_L \rangle \langle i_R | \sigma_\mu | j_R \rangle \\ &= A_8 d_8 [-\sin((\theta_b - \theta_i - \theta_k + \theta_j) / 2) \\ &\quad + \sin((\theta_b - \theta_i + \theta_k - \theta_j) / 2)]. \end{aligned} \quad (\text{A-28})$$

So the penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by

$$\begin{aligned} M_{(+1/2)}^{dip} &= A_8 d_8 \{ [-\sin((\theta_b + \theta_i - \theta_k - \theta_j) / 2) \\ &\quad + \sin((\theta_b - \theta_i + \theta_k - \theta_j) / 2)] \\ &\quad + [-\sin((\theta_b - \theta_i - \theta_k + \theta_j) / 2) \\ &\quad + \sin((\theta_b - \theta_i + \theta_k - \theta_j) / 2)] \}, \end{aligned} \quad (\text{A-29})$$

$$\begin{aligned} M_{(-1/2)}^{dip} &= A_8 d_8 \{ [-\cos((\theta_b - \theta_i - \theta_k + \theta_j) / 2) \\ &\quad - \cos((\theta_b - \theta_i + \theta_k - \theta_j) / 2)] \\ &\quad + [-\cos((\theta_b + \theta_i - \theta_k - \theta_j) / 2) \\ &\quad - \cos((\theta_b - \theta_i + \theta_k - \theta_j) / 2)] \}. \end{aligned} \quad (\text{A-30})$$

## Appendix B

### Decay rate of the effective Hamiltonian

The effective  $\Delta B = 1$  Hamiltonian at scale  $\mu = O(m_b)$  for tree plus penguin and including the electroweak penguin and the magnetic dipole term is

$$\begin{aligned} H_{eff}^{\Delta B=1} &= 2\sqrt{2}G_F \{ [d_{1c}(\mu)Q_1^c(\mu) + d_{2c}(\mu)Q_2^c(\mu)] \\ &\quad + [d_{1u}(\mu)Q_1^u(\mu) + d_{2u}(\mu)Q_2^u(\mu)] \\ &\quad - [\sum_{i=3}^{6,8} d_i(\mu)Q_i(\mu) + \sum_{i=7}^{10} d'_i(\mu)Q'_i(\mu)] \}, \end{aligned} \quad (\text{B-1})$$

here  $d_1, \dots, d_6, d'_7, \dots, d'_{10}$  are defined by eq. (26),  $d_{1,2c,u} = d_{1,2}(i = j = c, u)$  and index  $k$  refer to d or s. Using (A-22) and (A-23) one can obtain the matrix elements of the effective Hamiltonian operators. In the

first step, the tree plus penguin operators ( $Q_1, \dots, Q_6$ ) are chosen. All of the terms  $Q_1, Q_2, Q_3, Q_4$  have a form L-L handed but terms  $Q_1, Q_2$  have a form  $\langle i | \tilde{\sigma}^\mu | b \rangle_L \langle k | \tilde{\sigma}_\mu | j \rangle_L$  and terms  $Q_3, Q_4$  have a form  $\langle k | \tilde{\sigma}^\mu | b \rangle_L \langle i | \tilde{\sigma}_\mu | j \rangle_L$ . Consider  $i, k$  and  $j$  momenta in  $XZ$  plane, so according to (A-22), for the  $Q_1, Q_2$  and, we can write for b spin projection 1/2 and -1/2. Also terms  $Q_1, Q_2$  and  $Q_3, Q_4$  differ only by a minus, because

$$\begin{aligned} \sin((\theta_i + \theta_j - \theta_k) / 2) &= -\sin((\theta_k - \theta_j - \theta_i) / 2), \\ \sin((\theta_i - \theta_j - \theta_k) / 2) &= -\sin((\theta_k + \theta_j - \theta_i) / 2). \end{aligned} \quad (\text{B-2})$$

Terms  $Q_5, Q_6$  are of the form L-R handed,  $\langle i | \tilde{\sigma}^\mu | b \rangle_L \langle k | \sigma_\mu | j \rangle_R$ . The main forms of the terms  $Q_5, Q_6$  are  $\langle k | \tilde{\sigma}^\mu | b \rangle_L \langle i | \sigma_\mu | j \rangle_R$ . These terms can be written according to (A-23). So, the matrix element for b quark spin project 1/2 is given by

$$\begin{aligned} M_{eff} &= 2\sqrt{2}G_F \{ (A_1 + A_2) [\sin((\theta_k - \theta_j - \theta_i) / 2) \\ &\quad + \sin((\theta_k + \theta_j - \theta_i) / 2)] \\ &\quad - A_3 [\sin((\theta_k - \theta_i - \theta_j) / 2) \\ &\quad - \sin((\theta_k + \theta_i - \theta_j) / 2)] \}. \end{aligned} \quad (\text{B-3})$$

here  $A_1, A_2$  and  $A_3$  are combination of Wilson coefficients and colour factors. The forms of  $(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL}$  and  $(\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}$  are according to (A-22) and (A-23) respectively. So the square of spin average term  $Q_1, \dots, Q_6$  is given by

$$\begin{aligned} &[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]_{sp-av}^2 \\ &= \alpha_1 (1/16)(1+v_i)(1+v_k)(1+v_j) [1 - \cos(\theta_k - \theta_i)] \\ &\quad + \alpha_2 (1/16)(1-v_i)(1+v_k)(1-v_j) [1 + \cos(\theta_k - \theta_j)] \\ &\quad + \alpha_3 (1/16)\sqrt{1-v_i^2}(1+v_k)\sqrt{1-v_j^2} [1 + \cos(\theta_j - \theta_i) \\ &\quad - \cos(\theta_k - \theta_j) - \cos(\theta_k - \theta_i)]. \end{aligned} \quad (\text{B-4})$$

Now, one must obtain all of the helicity states for  $Q_1, \dots, Q_6$  and then sum them all. Adding eight terms of helicity states, on finds

$$\begin{aligned} &[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]_{sp-av}^2 = \\ &\quad (\alpha_1 / 2) [1 - v_i v_k \cos(\theta_k - \theta_i)] \\ &\quad + (\alpha_2 / 2) [1 + v_k v_j \cos(\theta_j - \theta_k)] \\ &\quad + (\alpha_3 / 2) \sqrt{1-v_i^2} \sqrt{1-v_j^2}. \end{aligned} \quad (\text{B-5})$$

After adding all colour combinations  $\alpha_1, \alpha_2$  and  $\alpha_3$  gives

$$\begin{aligned} \alpha_1 &= |d_1 + d_2 + d_3 + d_4|^2 \\ &\quad + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2, \end{aligned}$$

$$\begin{aligned}\alpha_2 &= |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2, \\ \alpha_3 &= \text{Re}\{(3d_1 + d_2 + d_3 + 3d_4)d_6^* \\ &\quad + (d_1 + 3d_2 + 3d_3 + d_4)d_5^*\},\end{aligned}\quad (\text{B-6})$$

here  $d_1, \dots, d_6$  are defined by eq. (26). The energy conservation gives

$$\begin{aligned}\cos(\theta_k - \theta_i) &= [(M_l - E_i - E_k)^2 \\ &\quad - (m_j^2 + p_i^2 + p_k^2)] / 2p_i p_k, \\ \cos(\theta_j - \theta_k) &= [(M_l - E_k - E_j)^2 \\ &\quad - (m_i^2 + p_k^2 + p_j^2)] / 2p_k p_j.\end{aligned}\quad (\text{B-7})$$

The angle between the particle velocities must be physical,  $-1 \leq \cos(\theta_k - \theta_i) \leq 1$  and

$-1 \leq \cos(\theta_j - \theta_k) \leq 1$ . So we should take the variable

$p_i$  and  $p_k$ , or  $x$  and  $y$  as,

$$p_i = xM_l / 2, \quad p_k = yM_l / 2 \quad (\text{B-8})$$

Then  $p_j$  is given by energy conservation

$$E_j = M_b - E_i - E_k = \sqrt{m_j^2 + p_j^2}. \quad \text{Also}$$

$$\cos(\theta_i - \theta_k) = (p_j^2 - p_i^2 - p_k^2) / 2p_i p_k, \quad \text{and so on.}$$

Momentum conservation gives

$$p_k \cos(\theta_i - \theta_k) + p_j \cos(\theta_j - \theta_i) = -p_i, \quad (\text{B-9})$$

and so on. Also

$$p_k \sin(\theta_i - \theta_k) = \pm p_j \sin(\theta_j - \theta_i). \quad (\text{B-10})$$

and so on. The partial decay rate, b spin averaged and summed over final spin states, has overall spherical symmetry. Apart from its overall orientation, a final state is specified by only two parameters, say  $p_i = |\mathbf{p}_i|$  and

$p_k = |\mathbf{p}_k|$ . The partial decay rate in the b rest frame is

$$\begin{aligned}d^2\Gamma_{Q_1, \dots, Q_6} / dp_i dp_k &= (G_F^2 / \pi^3) p_i p_k E_{\bar{j}} \{ \alpha_1 (p_i \cdot p_k / E_i E_k) \\ &\quad + \alpha_2 (p_i \cdot p_{\bar{j}} / E_i E_{\bar{j}}) + \alpha_3 (m_k m_{\bar{j}} / E_k E_{\bar{j}}) \},\end{aligned}\quad (\text{B-11})$$

here

$$p_i \cdot p_k = (M_b^2 + m_j^2 - m_i^2 - m_k^2 - 2M_b E_{\bar{j}}) / 2,$$

$$p_i \cdot p_{\bar{j}} = (M_b^2 + m_k^2 - m_i^2 - m_{\bar{j}}^2 - 2M_b E_k) / 2. \quad (\text{B-12})$$

After the change of variable to  $x$  and  $y$ , the decay rate is given by

$$d^2\Gamma_{Q_1, \dots, Q_6} / dx dy = \Gamma_{0b} I_{ps}^{EH}, \quad (\text{B-13})$$

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3, \quad (\text{B-14})$$

where

$$I_{ps}^1 = 6xy \cdot f_{ab} \cdot (1 - h_{abc}),$$

$$I_{ps}^2 = 6xy \cdot f_{bc} \cdot (1 + h_{bca}),$$

$$I_{ps}^3 = 6xy \cdot f_{ac} \cdot h_{xa} \cdot h_{yc}. \quad (\text{B-15})$$

Here

$$f_{ab} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + b^2},$$

$$h_{abc} = \frac{(f_{ab})^2 - (c^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + b^2}},$$

$$f_{bc} = 2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2},$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2 + x^2 + y^2)}{2\sqrt{x^2 + b^2}\sqrt{y^2 + c^2}},$$

$$f_{ac} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + c^2},$$

$$h_{acb} = \frac{(f_{ac})^2 - (b^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + c^2}}.$$

$$h_{xa} = [1 - (x^2 / (x^2 + a^2))]^{1/2},$$

$$h_{yc} = [1 - (y^2 / (y^2 + c^2))]^{1/2}, \quad (\text{B-16})$$

where  $a, b$  and  $c$  are:

$$a = 2m_i / M_b, \quad b = 2m_k / M_b, \quad c = 2m_j / M_b. \quad (\text{B-17})$$

## Appendix C

### Effective Hamiltonian of magnetic dipole decay rate

We calculate the decay rates of  $b \rightarrow q_i q_k \bar{q}_j$  according to

effective Hamiltonian ( $Q_1, \dots, Q_6$ ), including magnetic dipole ( $Q_8$ ) terms. The amplitude of Effective Hamiltonian for operators  $Q_1, Q_2, Q_3, Q_4$  is given by

$$\begin{aligned}M1_{(1/2)}^{L-L} &= A(d_1 + d_2 + d_3 + d_4) [\sin((\theta_k - \theta_j - \theta_i) / 2) \\ &\quad + \sin((\theta_k + \theta_j - \theta_i) / 2)], \\ M2_{(-1/2)}^{L-L} &= A(d_1 + d_2 + d_3 + d_4) [\cos((\theta_k - \theta_j - \theta_i) / 2) \\ &\quad - \cos((\theta_k + \theta_j - \theta_i) / 2)],\end{aligned}\quad (\text{C-1})$$

and for operators  $Q_5, Q_6$  is as well

$$\begin{aligned}M3_{(1/2)}^{L-R} &= A'(d_5 + d_6) [\sin((\theta_k - \theta_i - \theta_j) / 2) \\ &\quad - \sin((\theta_k + \theta_i - \theta_j) / 2)], \\ M4_{(-1/2)}^{L-R} &= A'(d_5 + d_6) [\cos((\theta_k - \theta_i - \theta_j) / 2) \\ &\quad + \cos((\theta_k + \theta_i - \theta_j) / 2)],\end{aligned}\quad (\text{C-2})$$

where  $d_1, \dots, d_6, d_8$  and  $A_8$  are defined by eq. (26) and (A-21) respectively and

$$\begin{aligned}A &= \sqrt{(1+v_i)}\sqrt{(1+v_k)}\sqrt{(1+v_{\bar{j}})}, \\ A' &= \sqrt{(1-v_i)}\sqrt{(1+v_k)}\sqrt{(1-v_{\bar{j}})},\end{aligned}\quad (\text{C-3})$$

Also the amplitude of magnetic dipole according to (A-20) is given by

$$\begin{aligned}M^{dip} &= A_8 d_8 [\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\ &\quad + \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle],\end{aligned}\quad (\text{C-4})$$

For various b spin project 1/2 and -1/2 the first term of (C-4) is given by

$$M5_{(1/2)}^{dip,L-L} = AA_8d_8[\sin((\theta_i - \theta_k + \theta_j)/2) + \sin((\theta_i + \theta_k - \theta_j)/2)],$$

$$M6_{(-1/2)}^{dip,L-L} = AA_8d_8[-\cos((\theta_i - \theta_k + \theta_j)/2) - \cos((\theta_i + \theta_k - \theta_j)/2)], \quad (C-5)$$

and the second term of (C-4) is given by

$$M7_{(1/2)}^{dip,L-R} = A'A_8d_8[\sin((\theta_i - \theta_k + \theta_j)/2) - \sin((\theta_i - \theta_k - \theta_j)/2)],$$

$$M8_{(-1/2)}^{dip,L-R} = A'A_8d_8[-\cos((\theta_i - \theta_k + \theta_j)/2) - \cos((\theta_i - \theta_k - \theta_j)/2)]. \quad (C-6)$$

Now the amplitude of  $Q_1, \dots, Q_6$  and  $Q_8$  (magnetic dipole term) should be added for b spine project 1/2 and -1/2, so

$$M_{(1/2)}^{tot} = M1_{(1/2)}^{L-L} + M3_{(1/2)}^{L-R} + M5_{(1/2)}^{dip,L-L} + M7_{(1/2)}^{dip,L-R},$$

$$M_{(-1/2)}^{tot} = M2_{(-1/2)}^{L-L} + M4_{(-1/2)}^{L-R} + M6_{(-1/2)}^{dip,L-L} + M8_{(-1/2)}^{dip,L-R}, \quad (C-7)$$

or

$$M_{(-1/2)}^{tot} = e_1 \cos((\theta_k - \theta_i - \theta_j)/2) - e_2 \cos((\theta_k - \theta_i + \theta_j)/2) + e_3 \cos((\theta_k + \theta_i - \theta_j)/2),$$

$$M_{(1/2)}^{tot} = e_1 \sin((\theta_k - \theta_i - \theta_j)/2) + e_2 \sin((\theta_k - \theta_i + \theta_j)/2) - e_3 \sin((\theta_k + \theta_i - \theta_j)/2), \quad (C-8)$$

here

$$e_1 = A[(d_1 + d_2 + d_3 + d_4) + A_8d_8] + A'[(d_5 + d_6) + A_8d_8],$$

$$e_2 = A(d_1 + d_2 + d_3 + d_4) - A'A_8d_8,$$

$$e_3 = AA_8d_8 + A'(d_5 + d_6). \quad (C-9)$$

The spin average of b spin project of 1/2 and -1/2 is given by

$$\left| M_{spin-ave}^{tot} \right|^2 = \frac{1}{2} \left[ \left| M_{(1/2)}^{tot} \right|^2 + \left| M_{(-1/2)}^{tot} \right|^2 \right]$$

$$= \frac{1}{2} [e_1^2 + e_2^2 + e_3^2 - 2e_1e_2 \cos(\theta_k - \theta_i) + 2e_1e_3 \cos(\theta_j - \theta_k) - 2e_2e_3 \cos(\theta_j - \theta_i)]. \quad (C-10)$$

After adding all color factors, it gives

$$e_1 = A[h_1 + h_2] + A'[h_2 + h_3], \quad e_2 = Ah_1 - A'h_2,$$

$$e_3 = Ah_2 + A'h_3, \quad (C-11)$$

where

$$h_1 = \sqrt{|d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2},$$

$$h_2 = A_8d_8, \quad h_3 = \sqrt{|d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2}, \quad (C-12)$$

The first term of (C-10) is given by

$$1) \quad e_1^2 + e_2^2 + e_3^2 = A^2(2h_1^2 + 2h_2^2 + 2h_1h_2) + A'(2h_2^2 + 2h_3^2 + 2h_2h_3) + 2AA'(h_2^2 + h_1h_3 + 2h_2h_3), \quad (C-13)$$

Adding eight terms of helicity states, so

$$e_1^2 + e_2^2 + e_3^2 = 8(g_1 + g_2 + 2\sqrt{1-v_i^2}\sqrt{1-v_j^2}g_3), \quad (C-14)$$

The second term of (C-10) is given by

$$2) \quad 2e_1e_2 \cos(\theta_k - \theta_i) = 2\{A^2(h_1^2 + h_1h_2) - A'^2(h_2^2 + h_2h_3) + AA'(h_1h_3 - h_2^2)\} \cos(\theta_k - \theta_i), \quad (C-15)$$

Adding eight terms of helicity states, so

$$2e_1e_2 \cos(\theta_k - \theta_i) = 2.8.v_k v_i (g_4 - g_5) \cos(\theta_k - \theta_i), \quad (C-16)$$

The third term of (C-10) is given by

$$3) \quad 2e_1e_3 \cos(\theta_j - \theta_k) = 2\{A^2(h_2^2 + h_1h_2) + A'^2(h_3^2 + h_2h_3) + AA'(h_1h_3 + 2h_2h_3 + h_2^2)\} \cos(\theta_j - \theta_k). \quad (C-17)$$

Adding eight terms of helicity states, so

$$2e_1e_3 \cos(\theta_j - \theta_k) = 2.8.v_j v_k (g_6 + g_7) \cos(\theta_j - \theta_k), \quad (C-18)$$

the fourth term of (C-10) is given by

$$4) \quad 2e_2e_3 \cos(\theta_j - \theta_i) = 2\{A^2(h_1h_2) - A'^2(h_2h_3) + AA'(h_1h_3 - h_2^2)\} \cos(\theta_j - \theta_i), \quad (C-19)$$

adding eight terms of helicity states, so

$$2e_2e_3 \cos(\theta_j - \theta_i) = 2.8.v_j v_i (g_8 - g_9) \cos(\theta_j - \theta_i), \quad (C-20)$$

here

$$g_1 = 2h_1^2 + 2h_2^2 + 2h_1h_2, \quad g_2 = 2h_2^2 + 2h_3^2 + 2h_2h_3,$$

$$g_3 = h_2^2 - h_1h_2 + h_1h_3 + 3h_2h_3, \quad g_4 = h_1^2 + h_1h_2,$$

$$g_5 = h_2^2 + h_2h_3, \quad g_6 = h_2^2 + h_1h_2, \quad g_7 = h_3^2 + h_2h_3,$$

$$g_8 = h_1h_2, \quad g_9 = h_2h_3, \quad (C-21)$$

The total amplitude of (C-10) is given by

$$\left| M_{spin-ave}^{tot} \right|^2 = \frac{1}{4} [g_1 + g_2 - 2\sqrt{1-v_i^2}\sqrt{1-v_j^2}g_3 - 2.v_k v_i (g_4 - g_5) \cos(\theta_k - \theta_i) + 2.v_j v_k (g_6 + g_7) \cos(\theta_j - \theta_k) - 2.v_j v_i (g_8 - g_9) \cos(\theta_j - \theta_i)], \quad (C-22)$$

or

$$\left| M_{spin-ave}^{tot} \right|^2 = \frac{1}{4} [g_1 - 2.v_k v_i (g_4 - g_5) \cos(\theta_k - \theta_i) + g_2 + 2.v_j v_k (g_6 + g_7) \cos(\theta_j - \theta_k) - 2\sqrt{1-v_i^2}\sqrt{1-v_j^2}g_3 - 2.v_j v_i (g_8 - g_9) \cos(\theta_j - \theta_i)], \quad (C-23)$$

Also one can obtain the amplitude of tree-level and Effective Hamiltonian ( $Q_1, \dots, Q_6$ ). The amplitude of tree-level ( $d_2 = d_3 = d_4 = d_5 = d_6 = d_8 = 0$ ) is given by

$$\begin{aligned} |M^{tot}|_{spin-ave, TL}^2 &= \frac{1}{4} [2h_1^2 - 2h_1^2 v_i v_k \cos(\theta_k - \theta_i) + 0 + 0] \\ &= 3d_1^2 \cdot \frac{1}{2} (1 - v_i v_k \cos(\theta_k - \theta_i)), \end{aligned} \quad (C-24)$$

and the amplitude of effective Hamiltonian ( $d_8 = 0$ ) is given by

$$\begin{aligned} |M^{tot}|_{spin-ave, EH}^2 &= (h_1^2 / 2) [1 - v_k v_i \cos(\theta_k - \theta_i)] \\ &\quad + (h_3^2 / 2) [1 + v_j v_k \cos(\theta_j - \theta_k)] \\ &\quad - (\sqrt{1 - v_i^2} \sqrt{1 - v_j^2} / 4) (2h_1 h_3). \end{aligned} \quad (C-25)$$

After integration in the phase space and changing the variables  $x$  and  $y$ ,

$$x = 2p_i / m_b, \quad y = 2p_k / m_b, \quad (C-26)$$

the differential of decay rate of effective Hamiltonian plus magnetic dipole (C-23) is given by

$$\frac{d^2\Gamma}{dx dy} = \frac{G_F^2 M_b^2}{192\pi^3} \frac{1}{2} (I_1 + I_2 + I_3), \quad (C-27)$$

where

$$\begin{aligned} I_1 &= 6xy f_{ab} [g_1 - 2(g_4 - g_5)h_{abc}], \\ I_2 &= 6xy f_{bc} [g_2 + 2(g_6 + g_7)h_{bca}], \\ I_3 &= 6xy f_{ac} [-2g_3 h_{xa} h_{yc} - 2(g_8 - g_9)h_{acb}], \end{aligned} \quad (C-28)$$

and  $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}, h_{yc}$  defined by (B-16).

## References

1. Z Xiao, C S Li, K T Chao, *Phys. Lett. B* **473** (2000) 148-156.
2. M Beneke, J Rohrer and D Yang, *Phys. Rev. Lett.* **96** (2006) 141801.
3. S A Abel, W N Cottingham and I B Wittingham, *Phys. Rev. D* **58** (1998) 073006.
4. A J Buras, R Fleischer, S Recksiegel, F Schwab, the *Proceedings of EPS-HEP* (2005).
5. C D Lu, Y L Shen, W Wang, *Chin. Phys. Lett.* **23** (2006) 2684-2687.
6. M Beneke, S Jager, *Nucl. Phys. B* **768** (2007) 51-84.
7. W N Cottingham, H Mehrban, I B Wittingham, *Phys. Rev. D* **60** (1999) 114029.
8. H N Li, *Czech. J. Phys.* **53** (2003) 657-666, *Prog. Part. Nucl. Phys.* **51** (2003) 85-171.
9. Z Xiao, W Li, L Guo, G Lu, *Eur. Phys. J. C* **18** (2001) 681-709.
10. A J Buras, P Gambino, U Haisch, *Nucl. Phys. B* **570** (2000) 117-154.
11. S Actis, G Passarino, *Equations and their Solutions*, hep-ph/0612124.
12. A J Buras, P Gambino, U Haisch, *Nucl. Phys. B* **570** (2000) 117-154.
13. J B Mendy, J Govaerts, To be published in the *Proceedings of the Second International Workshop on Contemporary Problems in Mathematical Physics*, Cotonou, Republic of Benin.
14. N G Deshpande, X G He, S Pakvasa, *Phys. Lett. B* **326** (1994) 307-311.
15. A J Buras, *Nucl. Instrum. Meth. A* **368** (1995) 1-20.
16. A Ali, C Greub, *Phys. Rev. D* **57** (1998) 2996-3016.
17. G Burdman, Ian. Shipsey, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 431-499.
18. A Datta, D London, *JHEP* **0404** (2004) 072.
19. Particle Data Group, *European Physical J. C* **3** (2007) 1.
20. H Y Cheng, hep-ph/9807323.
21. N G Deshpande, B Dutta, S Oh, *Phys. Rev. D* **57** (1998) 5723-5731.
22. H Y Cheng, B Tseng, *Phys. Rev. D* **58** (1998) 094005.
23. Y H Chen, H Y Cheng, B Tseng, *Phys. Rev. D* **59** (1999) 074003.