IRANIAN JOURNAL OF PHYSICS RESEARCH

Iranian Journal of Physics Research, Vol. 9, No. 4, 2010

The role of electroweak penguin and magnetic dipole QCD penguin on hadronic b Quark Decays

H Mehrban

Physics Department, Semnan University, P.O. Box 35195-363, Semnan, Iran
E-mail: hmehraban@semnan.ac.ir

(Received 16 March 2008 , in final from 24 September 2009)

Abstract

This research, works with the effective Hamiltonian and the quark model. Using, the decay rates of matter-antimatter of $\boldsymbol{b}$ quark was investigated. We described the effective Hamiltonian theory which was applied to the calculation of current-current ($Q_{1,2}$), QCD penguin ($Q_{3,...,6}$), magnetic dipole (Q_8) and electroweak penguin ($Q'_{7,...,10}$) decay rates. The gluonic penguin structure of hadronic $\boldsymbol{b}$ decays $b \to q_k g \to q_k q_i q_{\bar{j}}$ was studied through the Wilson coefficients of the effective Hamiltonian. The branching ratios of the Tree-Level, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole $\boldsymbol{b}$ quark decays $b \to q_i q_k q_{\bar{j}}$, $q_i \in \{u, c\}$, $q_k \in \{d, s\}$, $q_{\bar{j}} \in \{\bar{u}, \bar{c}\}$ have been calculated. It was shown that, the electroweak penguin and magnetic dipole contributions in b quark decays are small and current-current operators are dominated.

Keywords: b quark, QCD penguin, electroweak penguin, magnetic dipole

1. Introduction

Conservation of the gluonic current requires the $b \to q_k g$ vertex to have the structure [1,2]:

$$\Gamma^a_\mu(q^2) = (ig_s / 4\pi^2)\bar{u}_k(p_k)T^a V_\mu(q^2)u_b(p_b)\,, \tag{1}$$

where ,

$$V_\mu(q^2) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma^\nu [F_1^L(q^2)P_L + F_1^R(q^2)P_R] + i\sigma_{\mu\nu}q^\nu [F_2^L(q^2)P_L + F_2^R(q^2)P_R]\,. \tag{2}$$

Here F_1 and F_2 are respectively the electric (monopole) and magnetic (dipole) form factors, $q = q_g = p_b - p_k$ is the gluon four momentum, $P_{L(R)} \equiv (1 \mp \gamma_5)/2$ are the chirality projection operators and T^a $(a = 1,...,8)$ are the $SU(3)_c$ generators normalized to $Tr(T^a T^b) = \delta^{ab}/2$. The $\bar{b} \to \bar{q}_k g$ vertex is

$$\bar{\Gamma}^a_\mu(q^2) = -(ig_s / 4\pi^2)\bar{v}_b(p_b)T^a \bar{V}_\mu(q^2)v_k(p_k) \tag{3}$$

Here $\bar{V}_\mu$ has the form eq. (2) with the form factors $F_{1,2}^{L,R}(q^2)$ replaced by $\bar{F}_{1,2}^{L,R}(q^2)$. To lowest order in α_s the penguin amplitude for the decay process $b \to q_k g \to q_k q'\bar{q}'$ $(q_k q_i \bar{q}_j)_{i=j}$ is

$$M^{Peng} = -i(\alpha_s/\pi)[\bar{u}_k(p_k)T^a \Lambda_\mu u_b(p_b)] \times [\bar{u}_{q'}(p_{q'})\gamma^\mu T^a v_{\bar{q}'}(p_{\bar{q}'})], \tag{4}$$

where $\alpha_s = g_s^2/4\pi$ and

$$\Lambda_\mu \equiv \gamma_\mu [F_1^L(q^2)P_L + F_1^R(q^2)P_R] + (i\sigma_{\mu\nu}q^\nu / q^2) \times [F_2^L(q^2)P_L + F_2^R(q^2)P_R], \tag{5}$$

Similarly, for $\bar{b} \to \bar{q}_k q'\bar{q}'$, the amplitude is

$$\bar{M}^{Peng} = i(\alpha_s/\pi)[\bar{v}_k(p_k)T^a \bar{\Lambda}_\mu v_b(p_b)] \times [\bar{u}_{q'}(p_{q'})\gamma_\mu T^a v_{\bar{q}'}(p_{\bar{q}'})], \tag{6}$$

where $\bar{\Lambda}_\mu$ is obtained from eq. (5) by the replacement of all the $F(q^2)$ form factors by $\bar{F}(q^2)$ form factors. The top quark dominates in the sum for F_2 , hence at value of q^2 (a good approximation), we have $F_2^L(q^2) \approx F_2^L(0)$ and $F_2^R(q^2) \approx F_2^R(0)$ [3], so

$$F_1^L(q^2) = (G_F/\sqrt{2}) \sum_{i=u,c,t} V_{ik}^* V_{ib} f_1(x_i, q^2)\,,\ F_1^R(0) = 0\,, \tag{7}$$

$$F_2^L(0)/m_q = F_2^R(0)/m_b = (G_F/\sqrt{2}) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i)\,, \tag{8}$$

where $x_i \equiv m_i^2 / M_W^2$ $(i = u, c, t)$ and

$$f_2(x) = -(x/4(1-x)^4)[2+3x-6x^2+x^3+6x\ln x]\,, \tag{9}$$

$$f_1(x) = (1/12(1-x)^4) \times [18x - 29x^2 + 10x^3 + x^4 - (8 - 32x + 18x^2)\ln x], \tag{10}$$

$$f_1(x_i, q^2) = (10/9) - (2/3)\ln x_i + (2/3z_i) - (2(2z_i+1)/3z_i) g(z_i). \tag{11}$$

Here $z_i \equiv q^2 / 4m_i^2$ and

$$g(z) = \begin{cases} \sqrt{\dfrac{1-z}{z}} \arctan(\sqrt{\dfrac{z}{1-z}}), & z < 1 \\ \dfrac{1}{2}\sqrt{\dfrac{z-1}{z}}[\ln(\dfrac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}}) - i\pi], & z > 1 \end{cases} \tag{12}$$

For the u quark, z_i is large and we use the asymptotic form of eq. (11),

$$f_1(x_u, q^2) = (10/9) - (2/3)[\ln(q^2/M_W^2) - i\pi]\,. \tag{13}$$

We find $F_1^L >> F_1^R$ and $F_2^R >> F_2^L$. For the $b \to dq'\bar{q}'$ amplitude we find that F_1^L is dominant. Processes like $b \to ds\bar{s}$ and $\bar{b} \to \bar{d}s\bar{s}$ are expected to be penguin dominated [4] and F_1^L dominates all the other form factors. In the $b \to sq'\bar{q}'$ transition, we again find that $F_1^L >> F_1^R$, $F_2^R >> F_2^L$ and the F_1^L amplitude to be dominant.

Now, a very important issue is the generation of QCD corrections to penguin operators. Consider, for example, the local operator $(\bar{u}_\alpha b_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}$, which is directly induced by W-boson exchange. In this case, additional QCD correction diagrams, with a gluon contribute and as a consequence four operators, instead of two, are involved in the mixing under renormalization. These are [5, 6]:

$$Q_3 = (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}\,,$$

$$Q_4 = (\bar{d}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V-A}\,,$$

$$Q_5 = (\bar{d}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A}\,,$$

$$Q_6 = (\bar{d}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V+A}\,, \tag{14}$$

α and β are colour indices. The sum over q runs over all quark flavours that exist in the effective theory in question. Since the gluon coupling is of course flavor conserving, it is clear that penguins cannot be generated from the operator current due to the gluon coupling in the lower part. For convenience, this vector structure is decomposed into a (V-A) and a (V+A) part according to chiral representation,

$$(\bar{q}_{i\alpha} b_\alpha)_{V\mp A}(\bar{q}_{k\beta} q_{j\beta})_{V\mp A} = (\bar{q}_{i\alpha}\gamma^\mu((1\mp\gamma_5)/2)b_\alpha)(\bar{q}_{k\beta}\gamma_\mu((1\mp\gamma_5)/2)q_{j\beta}) \tag{15}$$

and in the terms of two component spinors are given by,

$$\bar{q}_{i\alpha}\gamma^\mu((1-\gamma_5)/2)b_\alpha = q_{i\alpha L}^\dagger \tilde{\sigma}^\mu b_{\alpha L}\,,$$

$$\bar{q}_{k\beta}\gamma_\mu((1-\gamma_5)/2)q_{j\beta} = q_{k\beta L}^\dagger \tilde{\sigma}_\mu q_{j\beta L}\,,$$

$$\bar{q}_{i\alpha}\gamma^\mu((1+\gamma_5)/2)b_\alpha = q_{i\alpha R}^\dagger \sigma^\mu b_{\alpha R}\,,$$

$$\bar{q}_{k\beta}\gamma_\mu((1+\gamma_5)/2)q_{j\beta} = q_{k\beta R}^\dagger \sigma_\mu q_{j\beta R}\,. \tag{16}$$

For each of these, two different colour forms arise due to the colour structure of the exchanged gluon. The amplitude (4) can be written [7],

$$M^{Peng} = -i(G_F/\sqrt{2})\{(\alpha_s(M_W/8\pi) \times [\sum_{i=u,c} V_{ib}V_{iq}^* f_1(x_i,q^2) + V_{tb}V_{tq}^* f_1(x_t)]\, Q_P + (1/2)\sum_{i=u,c,t} V_{ib}V_{iq}^* f_2(x_i) Q_8\}, \tag{17}$$

Q_8 is the chromomagnetic dipole operator:

$$Q_8 = 4\alpha_s^2 m_b [\bar{q}_{i\alpha}\sigma^{\mu\nu}(1+\gamma_5)T_{\alpha\beta}^a b_\beta] \times (q_\nu / q^2)[\bar{q}_{\bar{j}}\gamma_\mu T^a q_k], \tag{18}$$

Here

$$Q_P = Q_4 + Q_6 - (1/3)(Q_3 + Q_5)\,. \tag{19}$$

As a weak decay under the presence of the strong interaction B meson decays require special techniques [8]. The main tool to calculate such B meson decays is the effective Hamiltonian theory [9,10]. It is a two - step program, starting with an operator product expansion (OPE) and performing a renormalization group equation (RGE) analysis afterwards [10,11,12]. The necessary machinery has been developed over the last years.

The derivation starts as follows: If the kinematics of the decay are of the kind that the masses of the internal particle M_i are much larger than the external momenta P, $M_i^2 >> p^2$, then the heavy particle can be integrated out. This concept takes concrete form with the functional integral formalism. It means that the heavy particles are removed as dynamical degrees of freedom from the theory hence their fields do not appear in the (effective) Lagrangian anymore. Their residual effect lies in the generated effective vertices [13]. In this way, an effective low energy theory can be constructed from a full theory like the Standard Model [14]. A well known example is the four-Fermi interaction, where the W-boson propagator is made local for $M_W^2 >> q^2$ (q denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2-M_W^2)\to ig_{\mu\nu}[(1/M_W^2)+(q^2/M_W^4)+...], \tag{20}$$

where the ellipses denote terms of higher order in $1/M_W$.

Apart from the t quark, the basic framework for weak decays quarks is the effective field theory relevant for scales $M_W, M_Z, M_t >> \mu$ [9,15]. This framework, as stated before, brings in local operators, which govern "effectively" the transition in question. From decaying quark point of view, it represents the generalization of the Fermi theory as formulated by Sudershan and Marshak and Feynman and Gell-Mann forty years ago.

It is well known that the decay amplitude is the product of two different parts, whose phases are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. Indeed the simplest effective Hamiltonian without QCD effects ($b\to ud\bar{u}$) is

$$H_{eff}^0=2\sqrt{2}G_F\ V_{ub}V_{ud}^*\ Q_1, \tag{21}$$

where G_F is the Fermi constant, V_{ij} are the relevant CKM factors and

$$Q_1=(\bar{u}_\alpha b_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}, \tag{22}$$

is a $(V-A)$, $(V-A)$ is current-current local operator. This simple tree amplitude introduces a new operator Q_2 and is modified by the QCD effect to

$$H_{eff}=2\sqrt{2}G_F\ V_{ub}V_{ud}^*(C_1Q_1+C_2Q_2), \tag{23}$$

Here

$$Q_2=(\bar{u}_\beta b_\alpha)_{V-A}(\bar{d}_\alpha u_\beta)_{V-A}, \tag{24}$$

where C_1 and C_2 are the Wilson coefficients. The situation in the Standard Model is, however, more complicated because of the presence of additional interactions in particular penguins which effectively generate new operators. These are in particular the gluon, photon and Z^0-boson exchanges and penguin b quark contributions.

Consequently, the relevant effective Hamiltonian for B-meson decays generally involves several operators Q_i with various colour and Dirac structures which are different from Q_1. The operators can be grouped into three categories [16]: $i=1,2-$ current-current operators, $i=3,...,6,8-$ gluonic penguin operators and $i=7',...,10'-$ Electroweak penguin operators. Moreover, each operator is multiplied by a calculable Wilson coefficient $C_i(\mu)$:

$$H_{eff}=2\sqrt{2}G_F[\sum_{i=1}^{6,8}d_i(\mu)Q_i(\mu)+\sum_{i=7}^{10}d_i'(\mu)Q_i'(\mu)], \tag{25}$$

where the scale μ is discussed below, $d_i(\mu)=V_{CKM}C_i(\mu)$ and V_{CKM} denotes the relevant CKM factors that are:

$$d_{1,2}=V_{ib}V_{jk}^*C_{1,2},$$
$$d_{3,...,6}=-V_{tb}V_{tk}^*C_{3,...,6},$$
$$d_8=-(\alpha_s/4\pi)V_{tb}V_{tk}^*C_8,$$
$$d'_{7,...,10}=-(3/2)e_q\ V_{tb}V_{tk}^*C_{7,...,10}. \tag{26}$$

For tree-level the $d_{1,2}$ coefficients are:

$$d_1=V_{ib}V_{kj}^*,\ \ d_2=0. \tag{27}$$

and for the effective penguin model the $d_{3,...,6,8}$ coefficients are:

$$d_3=d_5=(1/6)(\alpha_s/4\pi)\sum_{i=u,c,t}V_{iq}^*V_{ib}f_1(x_i),$$
$$d_4=d_6=-3d_3,$$
$$d_8=-(m_b/2)(\alpha_s/4\pi)\sum_{i=u,c,t}V_{iq}^*V_{ib}f_2(x_i). \tag{28}$$

At this stage it should be mentioned that the usual Feynman diagram containing full W propagators, Z^0 propagators and top quark propagators really represent the happening at scales $O(M_W)$ whereas the true picture of a decaying hadron is more correctly described by the local operators in question. Thus, whereas at scale $O(M_W)$, we have to deal with the full six-quark theory containing the photon, weak gauge bosons and gluon, at scale O(1GeV) the relevant effective theory contains only three light quarks u, d, and s, gluons and the photon. At intermediate energy scales $\mu=O(m_b)$ and $\mu=O(m_c)$ relevant for beauty and charm decays, effective five-quark and effective four-quark theories have to be considered, respectively [17].

The usual procedure then is to start at a high energy scale $O(M_W)$ and consecutively integrate out the heavy degrees of freedom (heavy with respect to the relevant scale μ) from explicitly appearing in the theory. The word explicitly is very essential here. The heavy fields did not disappear. Their effects are merely hidden in the effective gauge coupling constants, running masses and most importantly the coefficients describing the effective strength of the operators at a scale μ, the Wilson coefficient functions $C_i(\mu)$ [5,9,10,18]. It is straightforward to apply H_{eff} to B- and D-meson decays as well by changing the quark flavours appropriately. μ is some low-energy scale of O(1GeV), $O(m_c)$ and $O(m_b)$ for K, D, and B meson decays, respectively. The argument μ of the operators $Q_i(\mu)$ means that their matrix elements are to be normalized at scale μ.

2. Magnetic dipole amplitude of $b\to q_iq_k\bar{q}_j$

A charge particle in orbital motion generates a magnetic dipole moment of a magnitude proportional to its orbital angular momentum. Further more, a particle with intrinsic angular momentum or spin has an intrinsic magnetic moment. The magnetic dipole term in the penguin amplitude, according to eq. (5), is

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu}q^\nu / q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \qquad (29)$$

Also, according to eq. (8) magnetic (dipole) form factor at $q^2 = 0$ $(q^2 / M_W^2 << 1)$ is

$$\frac{F_2^L(0)}{m_k} = \frac{F_2^R(0)}{m_b} = \frac{g_2^2}{8M_W^2}\sum_i V_{ik}^* V_{ib} f_2(x_i). \qquad (30)$$

The top quark is dominant for $F_2^R(0)$, so we can write

$$F_2^R(0) = m_b(G_F / \sqrt{2})(V_{tk}^* V_{tb}) f_2(x_t). \qquad (31)$$

Here $f_2(x_t)$ defined by (9) and $x_t = m_t^2 / M_W^2$, also we saw that $F_2^L(0) << F_2^R(0)$, because $m_k << m_b$ so the magnetic dipole term becomes:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu}q^\nu / q^2)F_2^R(0)P_R. \qquad (32)$$

Putting in the penguin am really plitude, according to eq. (4),

$$M^{dip} = \frac{g_s^2}{4\pi^2} \times[\bar{u}_k(p_k)T^a(i\sigma_{\mu\nu}q^\nu / q^2)F_2^R(0)P_R u_b(p_b)] \times[\bar{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})]. \qquad (33)$$

The magnetic dipole of penguin amplitude is given by (see App.A),

$$M^{dip} = A_8 d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle], \qquad (34)$$

here

$$d_8 = -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^* V_{ib} f_2(x_i),$$

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2). \qquad (35)$$

Now we must calculate each terms of the above equation for b spins project -1/2 and 1/2, then squaring these terms and summing up all of them and finally, averaging. The penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by (see App.A),

$$\begin{aligned} M_{(+1/2)}^{dip} = A_8 d_8\{&[-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ &+ \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ &+[-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ &+ \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)]\}, \end{aligned} \qquad (36)$$

$$\begin{aligned} M_{(-1/2)}^{dip} = A_8 d_8\{&[-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ &- \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ &+[-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ &- \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)]\}. \end{aligned} \qquad (37)$$

3. Effective Hamiltonian decay rates of $b \to q_i q_k \bar{q}_j$

The effective $\Delta B = 1$ Hamiltonian at scale $\mu = O(m_b)$ for tree plus penguin and including the electroweak penguin and the magnetic dipole term is [5,6,9],

$$\begin{aligned} H_{eff}^{\Delta B=1} = 2\sqrt{2}G_F\{&[d_{1c}(\mu)Q_1^c(\mu) + d_{2c}(\mu)Q_2^c(\mu)] \\ &+[d_{1u}(\mu)Q_1^u(\mu) + d_{2u}(\mu)Q_2^u(\mu)] \\ &-[\sum_{i=3}^{6,8} d_i(\mu)Q_i(\mu) + \sum_{i=7}^{10} d_i'(\mu)Q_i'(\mu)]\}. \end{aligned} \qquad (38)$$

Here $d_1,...,_6, d_7',...,_{10}$ are defined by (26), $d_{1,2c,u} = d_{1,2}(i = j = c,u)$ and index k refer to d or s. The decay rate is given by (see App.B)

$$d^2\Gamma_{Q_1,...,Q_6} / dxdy = \Gamma_{0b} I_{ps}^{EH}, \qquad (39)$$

here

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3, \qquad (40)$$

where

$$I_{ps}^1 = 6xy.f_{ab}.(1 - h_{abc}),$$

$$I_{ps}^2 = 6xy.f_{bc}.(1 + h_{bca}),$$

$$I_{ps}^3 = 6xy.f_{ac}.h_{xa}.h_{yc}. \qquad (41)$$

and

$$\begin{aligned} \alpha_1 &= |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2, \\ \alpha_2 &= |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2), \\ \alpha_3 &= \mathrm{Re}\{(3d_1 + d_2 + d_3 + 3d_4)\, d_6^* \\ &\quad + (d_1 + 3d_2 + 3d_3 + d_4)\, d_5^*\}. \end{aligned} \qquad (42)$$

Here Γ_{0b}, f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{xa} and h_{yc} defined in Appendix B.

3. 1. Tree-Level

first of all, we obtain the decay rates of tree-level, without QCD corrections, from eq. (39) by choosing $d_1 = V_{ib}V_{jk}^*$ and $d_2 = d_3 = ... = d_6 = 0$, so $\alpha_1 = 3d_1^2, \alpha_2 = 0$ and $\alpha_3 = 0$. Q_1 is the conventional four-Fermi interaction operator thus eq. (39) reduces to $\Gamma_{Q_1,...,Q_6} = \Gamma_{0b} I_{ps}^{EH}$ ($I_{ps}^{EH} = \alpha_1 I_{ps}^1$), so

$$\Gamma_{tree} = 3\Gamma_{0b}\left|V_{ib}V_{jk}^*\right|^2 I_{ps}^1. \qquad (43)$$

3. 2. Pure Penguin

one can obtain the decay rates of pure penguin, by selecting $d_1 = d_2 = 0$, so that α_1 and α_3 reduce to,

$$\alpha_1 = |d_3 + d_4|^2 + 2|d_3|^2 + 2|d_4|^2,$$

$$\alpha_3 = \mathrm{Re}\{(d_3 + 3d_4)d_6^* + (3d_3 + d_4)d_5^*\}. \qquad (44)$$

α_2 and the decay rate keep the form eq. (42).

4. Effective Hamiltonian of electroweak penguin decay rates

The generalization of the $\Delta B = 1$ Hamiltonian in pure QCD to incorporate electroweak penguin operators is the sum of the $Q_1,...,Q_6,Q'_7,...,Q'_{10}$ eq. (38). The $\Delta B = 1$ Wilson coefficients for $Q_1^{u,c}, Q_2^{u,c}, Q_3,...,Q_6, Q'_7,...,Q'_{10}$ in the mixed case of QCD and QED. Therefore the discussion of $C_1,...,C_6$ is also valid for the present case. It was observed that, all of the terms Q_1,Q_2,Q_3,Q_4 had a form Left-Left handed. Terms Q_1,Q_2 have a form $\langle i|_L \tilde{\sigma}^\mu |b\rangle_L \langle k|_L \tilde{\sigma}_\mu |j\rangle_L$, and terms Q_3,Q_4 have a form $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_L \tilde{\sigma}_\mu |j\rangle_L$. Also terms Q_5,Q_6 have a form Left-Right handed, $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_R \sigma_\mu |j\rangle_R$. We consider that terms Q'_7,Q'_8 have a form Left-Right handed and terms Q'_9,Q'_{10} have a form Left-Left handed. Therefore terms Q'_7,Q'_8 have a form $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_L \tilde{\sigma}_\mu |j\rangle_L$, and terms Q'_9,Q'_{10} have a form $\langle k|_L \tilde{\sigma}^\mu |b\rangle_L \langle i|_R \sigma_\mu |j\rangle_R$. Thus the partial decay rate including electroweak penguin is the same as eq. (39) with different constants α_1,α_2 and α_3,

$$\alpha_1 = |d_1+d_2+d_3+d_4+d'_9+d'_{10}|^2 + 2|d_1+d_4+d'_{10}|^2 + 2|d_2+d_3+d'_9|^2,$$

$$\alpha_2 = |d_5+d_6+d'_7+d'_8|^2 + 2|d_5+d'_8|^2 + 2|d_6+d'_7|^2),$$

$$\alpha_3 = \mathrm{Re}\{(3d_1+d_2+d_3+3d_4+d'_9+3d'_{10})(d_6^*+d_7'^*) + (d_1+3d_2+3d_3+d_4+3d'_9+d'_{10})(d_5^*+d_8'^*)\}, \quad (45)$$

where $d_1,...,d_6,d'_7,...,d'_{10}$ defined by eq. (26) and e_q is the quark electric charge which is given by

$$e_{u,c,t} = 2/3,\ e_{d,s,b} = -(1/3). \quad (46)$$

5. Effective Hamiltonian of magnetic dipole decay rates

Now the aim is to calculate the decay rates of $b \to q_i q_k \bar{q}_j$ according to the effective Hamiltonian ($Q_1,...,Q_6$), including magnetic dipole (Q_8) terms. We obtained the amplitude of operators $Q_1,...,Q_6$, and the amplitude of magnetic dipole (Q_8) terms. Adding the amplitudes, we calculated the decay rates of operators of the effective Hamiltonian including magnetic dipole terms ($Q_1,...,Q_6,Q_8$). The amplitude of the effective Hamiltonian including magnetic dipole is given by (see App.C)

$$\left|M^{tot}\right|^2_{spin-ave} = \frac{1}{4}[g_1 - 2.v_k v_i (g_4-g_5)\cos(\theta_k-\theta_i) + g_2 + 2.v_j v_k (g_6+g_7)\cos(\theta_j-\theta_k) - 2\sqrt{1-v_i^2}\sqrt{1-v_j^2}\, g_3 - 2.v_j v_i (g_8-g_9)\cos(\theta_j-\theta_i)], \quad (47)$$

In order to check the above equation, one can obtain the amplitude of tree-level and the effective Hamiltonian $(Q_1,...,Q_6)$. The amplitude of tree-level ($d_2=d_3=d_4=d_5=d_6=d_8=0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,TL} = \frac{1}{4}[2h_1^2 - 2.h_1^2 v_i v_k \cos(\theta_k-\theta_i) + 0 + 0] = 3d_1^2.\frac{1}{2}(1 - v_i v_k \cos(\theta_k-\theta_i)), \quad (48)$$

and the amplitude of the effective Hamiltonian ($d_8 = 0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,EH} = (h_1^2/2)[1 - v_k v_i \cos(\theta_k-\theta_i)] + (h_3^2/2)[1 + v_j v_k \cos(\theta_j-\theta_k)] - (\sqrt{1-v_i^2}\sqrt{1-v_j^2}/4)(2h_1h_3). \quad (49)$$

The differential of decay rate of the effective Hamiltonian plus magnetic dipole eq. (47) is given by

$$\frac{d^2\Gamma}{dxdy} = \frac{G_F^2 M_b^2}{192\pi^3}\frac{1}{2}(I_1+I_2+I_3), \quad (50)$$

where

$$I_1 = 6xyf_{ab}\,[g_1 - 2(g_4-g_5)h_{abc}],$$
$$I_2 = 6xyf_{bc}\,[g_2 + 2(g_6+g_7)h_{bca}],$$
$$I_3 = 6xyf_{ac}\,[-2g_3 h_{xa}h_{yc} - 2(g_8-g_9)h_{acb}] \quad (51)$$

and $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}, h_{yc}$ defined by (B-16). Now the decay rates of eq. (50) is checked for Tree-Level and effective Hamiltonian $(Q_1,...,Q_6)$. Putting $d_2=d_3=d_4=d_5=d_6=d_8=0$ in eq. (50) the decay rate of Tree-Level is obtained which is the same as eq. (43) and putting $d_8=0$ in eq. (33) the decay rate of the effective Hamiltonian is found to be the same as eq. (39).

6. Effective Hamiltonian of electroweak penguin and magnetic dipole decay rates

The partial decay rate of the effective Hamiltonian plus Magnetic Dipole was given by eq. (50). Now the partial decay rate including electroweak penguin is obtained. In this case, the electroweak penguin operators (d'_7,d'_8,d'_9,d'_{10}) should be included in the as did before for obtaining the partial decay rate is given by

$$\frac{d^2\Gamma}{dxdy} = \frac{G_F^2 M_b^2}{192\pi^3}\frac{1}{2}(I_1+I_2+I_3), \quad (52)$$

where I_1,I_2,I_3 defined by eq. (51) and h_1,h_2,h_3 defined by,

Table 1. Effective Wilson coefficients C_i^{eff} at the renormalization scale $\mu = 2.5$ for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions.

	$b \to d$	$\bar{b} \to \bar{d}$	$b \to s$	$\bar{b} \to \bar{s}$
C_1^{eff}	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i
C_2^{eff}	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i
C_3^{eff}	0.0217+0.0018i	0.0234+0.0047i	0.0232+0.0030i	0.0231+0.0029i
C_4^{eff}	-0.0498-0.0054i	-0.0543-0.0142i	-0.0535-0.0091i	-0.0531-0.0086i
C_5^{eff}	0.0156+0.0018i	0.0173+0.0047i	0.0171+0.0030i	0.0170+0.0029i
C_6^{eff}	-0.0625-0.0054i	-0.0678-0.0142i	-0.0670-0.0091i	-0.0667-0.0086i

Table 2. Branching ratios (BR) of tree-level $b \to q_i q_k \bar{q}_j$ (43) ($\Gamma_{tot} = 3.0457 \times 10^{-13}$GeV).

Process	$BR \times 10^{-2}$	Process	$BR \times 10^{-2}$
$b \to ce^- \bar{\nu}_e$	14.62	$b \to ue^- \bar{\nu}_e$	0.231
$b \to c\mu^- \bar{\nu}_\mu$	14.62	$b \to u\mu^- \bar{\nu}_\mu$	0.231
$b \to c\tau^- \bar{\nu}_\tau$	0.714	$b \to u\tau^- \bar{\nu}_\tau$	0.084
$b \to cd\bar{u}$	49.02	$b \to ud\bar{u}$	0.725
$b \to cs\bar{c}$	16.13	$b \to ud\bar{c}$	0.019
$b \to cd\bar{c}$	0.857	$b \to us\bar{u}$	0.531
$b \to cs\bar{u}$	2.352	$b \to us\bar{c}$	0.355

$$h_1 = \sqrt{\begin{array}{l} |d_1 + d_2 + d_3 + d_4 + d'_9 + d'_{10}|^2 \\ + 2|d_1 + d_4 + d'_{10}|^2 + 2|d_2 + d_3 + d'_9|^2 \end{array}}$$

$$h_2 = A_8 d_8 ,$$

$$h_3 = \sqrt{|d_5 + d_6 + d'_7 + d'_8|^2 + 2|d_5 + d'_8|^2 + 2|d_6 + d'_7|^2} , \tag{53}$$

7. Numerical results

As an example of the use of the above formalism, we use the standard Particle Data Group [19] parameterization of the CKM matrix with the central values

$\theta_{12} = 0.221$, $\theta_{13} = 0.0035$, $\theta_{23} = 0.041$,

and choose the CKM phase δ_{13} to be $\pi/2$. Following Ali and Greub [16], we treat internal quark masses in tree-level loops with the values (GeV) $m_b = 4.88$, $m_s = 0.2$, $m_d = 0.01$, $m_u = 0.005$, $m_c = 1.5$, $m_e = 0.0005$, $m_\mu = 0.1$, $m_\tau = 1.777$ and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$. The effective Wilson coefficients C_i^{eff} at the renormalization scale $\mu = 2.5$GeV for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions, shown in the Table 1 [7]. Also, following H.Y.Cheng [20], [21,5,10] and [22, 23, 5, 10], we choose the effective Wilson coefficients of $C_7^{eff} - C_{10}^{eff}$,

$C_7^{eff} = -(0.0276 + i0.0369)\alpha_e$, $C_8^{eff} = 0.054\alpha_e$,

$C_9^{eff} = -(1.138 + i0.0369)\alpha_e$, $C_{10}^{eff} = 0.263\alpha_e$,

here $\alpha_e = 1/137$, that is the electromagnetic coupling constant. In order to obtain the total rates at the tree-level a sum is made over the b-quark decay rates. The total decay rate and branching ratios of several of semileptonic and hadronic modes are given in Table 2. We see that the modes $b \to cl\nu, b \to cd\bar{u}$ and $b \to cs\bar{c}$ are dominant. The total b-quark decay rate at the tree-level is given by

$\Gamma^T_{total} = \Gamma^T_{Semileptonic} + \Gamma^T_{Hadronic} = 3.0457 \times 10^{-13}$GeV.

It is observed that the decay rate for the antiparticle $\bar{b} \to \bar{u}\bar{d}u$ is greater than the particle decay rate $b \to ud\bar{u}$, and the antiparticle decay rate $\bar{b} \to \bar{c}\bar{d}c$ is less than the particle decay rate $b \to cd\bar{c}$, and so on. We consider that the modes $b \to c\bar{u}d$, and $b \to c\bar{c}s$ are dominant.

The branching ratios of the effective Hamiltonian $(Q_1,...,Q_6)$ for particles and antiparticles are collected in Table (3). In this case modes $b \to cd\bar{u}$ and $b \to cs\bar{c}$ are dominant. Also, the branching ratios including the

Table 3. Branching ratios ($BR\times10^{-2}$) of tree-level (T) eq. (43), Effective Hamiltonian (EH) eq. (39), Effective Hamiltonian including Electroweak Penguin $(EH+EP)$ eq. (45), Effective Hamiltonian including Magnetic Dipole $(EH+MD)$ eq. (50) and Effective Hamiltonian including Electroweak Penguin and Magnetic Dipole $(EH+EP+MD)$ eq. (52) of the particles and antiparticles for the various $b\to q$ $(\overline{b}\to\overline{q})$ transitions. The total decay rates are in unit of $10^{-13}\mathrm{GeV}$.

$\Gamma_{tot}\to$	3.0457	3.404	3.497	3.529	3.637
Process	(T)	$(E-H)$	$(EH+EP)$	$(EH+MD)$	$(EH+EP+MD)$
$b\to ud\overline{u}$	0.725	0.899	0.872	0.903	0.914
$b\to cd\overline{c}$	0.857	0.995	0.956	0.992	1.102
$b\to cd\overline{u}$	49.02	49.83	48.51	50.35	52.26
$b\to ud\overline{c}$	0.019	0.024	0.024	0.019	0.038
$b\to us\overline{u}$	0.531	0.603	0.544	0.680	0.714
$b\to cs\overline{c}$	16.13	17.09	18.19	17.780	18.69
$b\to us\overline{c}$	0.355	0.658	0.578	0.697	0.746
$b\to cs\overline{u}$	2.352	2.623	3.716	0.521	3.962
$\overline{b}\to\overline{u}\overline{d}u$	0.725	0.943	0.915	0.951	0.988
$\overline{b}\to\overline{c}\overline{d}c$	0.857	0.987	0.958	1.124	1.658
$\overline{b}\to\overline{c}\overline{d}u$	49.02	49.80	48.49	50.31	50.87
$\overline{b}\to\overline{u}\overline{d}c$	0.019	0.023	0.022	0.032	0.045
$\overline{b}\to\overline{u}\overline{s}u$	0.531	0.595	0.539	0.675	0.721
$\overline{b}\to\overline{c}\overline{s}c$	16.13	17.10	18.18	17.791	18.38
$\overline{b}\to\overline{u}\overline{s}c$	0.355	0.655	0.577	0.694	0.762
$\overline{b}\to\overline{c}\overline{s}u$	2.352	2.623	3.707	2.512	4.130

electroweak penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ are shown as well and those of the pure Penguin are given in Table 3 and 4 respectively. It is seen that, in the pure Penguin decays, modes $b\to ss\overline{s}$ and $b\to sd\overline{d}$ are dominant. Also, it is observed that, terms of Current-Current plus Penguin operators dominate as compared with the electroweak Penguin operators.

The branching ratios of the effective Hamiltonian plus magnetic dipole, and the effective Hamiltonian plus electroweak Penguin plus magnetic dipole are shown in Table 3 as well. It shows that the electroweak Penguin plus magnetic dipole term is small and can be neglected in the total decay rate. The total decay rate of the effective Hamiltonian $(Q_1,...,Q_6)$ of particle and antiparticle are given by

$$\Gamma^{EH}_{total(Q_1,...,Q_6)}=3.404\times10^{-13}\mathrm{GeV}.$$

In addition, the total decay rate of particles and antiparticles including electroweak penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ for b-quark decays are given by

$$\Gamma^{EH+EP}_{total(Q_1,...,Q_{10})}=3.497\times10^{-13}\mathrm{GeV}.$$

Also the total decay rate of particles and antiparticles including effective Hamiltonian and magnetic dipole $(Q_1,...,Q_6,Q_8)$ and the effective Hamiltonian, electroweak penguin and Magnetic Dipole $(Q_1,...,Q_6,Q_8,Q'_7,...,Q'_{10})$ for b-quark decays are given by

$$\Gamma^{EH+MD}_{total(Q_1,...,Q_8)}=3.526\times10^{-13}\mathrm{GeV},$$

$$\Gamma^{EH+EP+MD}_{total(Q_1,...,Q_8,...,Q_{10})}=3.637\times10^{-13}\mathrm{GeV}.$$

The total decay rates of pure penguin mode particles and antiparticles is

$$\Gamma^{P}_{total(Q_3,...,Q_6)}=2.479\times10^{-15}\mathrm{GeV}.$$

It is seen that for pure penguin modes, the decay rates of particles are less than those of antiparticles (see Table 4). It is interesting if a comparison is made among the decay rate of the Tree-Level (T) (see eq. (43)), effective Hamiltonian $(Q_1,...,Q_6)$ (EH) (see eq. (39)), effective Hamiltonian plus electroweak Penguin $(Q_1,...,Q_6,Q'_7,...,Q'_{10})$ $(EH+EP)$ (see (45)), effective Hamiltonian plus magnetic dipole $(EH+MD)$ (see eq. (50)) and the effective Hamiltonian plus electroweak penguin plus magnetic dipole $(Q_1,...,Q_6,Q_8,Q'_7,...,Q'_{10})$ $(EH+EP+MD)$ (see eq. (52)) that show at Table 3.

8. Conclusions

We obtained the decay rates of the b-quark at the tree-level, penguin, effective Hamiltonian, effective Hamiltonian including electroweak penguin, and for the first time, the effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole terms of the particles and antiparticles for the various

Table 4. Branching ratios of pure penguin of effective Hamiltonian of the particles and antiparticles for the various $b \to q$ $(\bar{b} \to \bar{q})$ transitions (44). ($\Gamma_{tot} = 3.404\times10^{-13}\text{GeV}$).

Process	$BR\times10^{-4}$	Process	$BR\times10^{-4}$
$b \to dd\bar{d}$	2.408	$\bar{b} \to \bar{d}\bar{d}d$	2.615
$b \to ds\bar{s}$	2.687	$\bar{b} \to \bar{d}\bar{s}s$	2.763
$b \to sd\bar{d}$	53.872	$\bar{b} \to \bar{s}\bar{d}d$	54.053
$b \to ss\bar{s}$	54.517	$\bar{b} \to \bar{s}\bar{s}s$	53.782

$b \to q$ $(\bar{b} \to \bar{q})$ transitions. According to Table 2, the dominant mode in b-quark in the semileptonic and hadronic decays are, $b \to c\ell^-\bar{\nu}_\ell$ $(\ell \to e,\mu)$ and $b \to cd\bar{u}$ respectively because the decay rates of $b \to c$ channel are very much bigger than those of $b \to u$, since $V_{cb} >> V_{ub}$. In addition, the dominant mode in the pure penguin decays is, $b \to s$. According to Table 4, the branching ratios of pure penguin of the effective Hamiltonian of the particles and antiparticles are close.

The electroweak penguin and magnetic dipole terms are small for b-quark decay rates (electroweak corrections and the magnetic dipole contributions are small) and the decay rate of the tree, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole of the particles and antiparticles are also not very different (see Table 3). The decay rates of $b-$ and $\bar{b}-$quark, at the tree-level are exactly the same, but in the pure penguin, effective Hamiltonian, effective Hamiltonian including electroweak penguin, effective Hamiltonian including magnetic dipole and the effective Hamiltonian including electroweak penguin and magnetic dipole, are different. For example, $\Gamma_{b\to sd\bar{d}} < \Gamma_{\bar{b}\to\bar{s}\bar{d}d}$, $\Gamma_{b\to ud\bar{u}} < \Gamma_{\bar{b}\to\bar{u}\bar{d}u}$, $\Gamma_{b\to cd\bar{u}} > \Gamma_{\bar{b}\to\bar{c}\bar{d}u}$ and $\Gamma_{b\to cs\bar{c}} \approx \Gamma_{\bar{b}\to\bar{c}\bar{s}c}$, because the total decay rates of $b-$ and $\bar{b}-$quark must be equal, $\Gamma_b^{total} = \Gamma_{\bar{b}}^{total}$.

Also the decay rates and branching ratios are very similar in all the models but the effective Hamiltonian including electroweak penguin and magnetic dipole total decay rate is about 10% larger than the simple tree or effective Hamiltonian. On the other hand, including the penguin term induced matter antimatter asymmetries. These are largest in the rare decays $b \to ud\bar{u}$, the decay rate of which, is about 7% smaller than that of $\bar{b} \to \bar{u}\bar{d}u$. Also the rate $b \to su\bar{u}$ is larger than the rate $\bar{b} \to \bar{s}\bar{u}u$.

Appendix A

Penguin amplitude of magnetic dipole

According to eq. (4) the penguin amplitude is given by

$$M^{dip} = \frac{g_s^2}{4\pi^2}[\bar{u}_k(p_k)T^a(i\sigma_{\mu\nu}q^\nu/q^2)F_2^R(0)P_R u_b(p_b)] \times[\bar{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})], \tag{A-1}$$

where

$$\sigma^{\mu\nu} = (i/2)[\gamma^\mu,\gamma^\nu] = (i/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \tag{A-2}$$

and

$$\gamma^\mu\gamma^\nu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0 \end{pmatrix} = \begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu \end{pmatrix}. \tag{A-3}$$

So

$$\sigma^{\mu\nu} = \frac{i}{2}\begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu - \sigma^\nu\tilde{\sigma}^\mu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu - \tilde{\sigma}^\nu\sigma^\mu \end{pmatrix}. \tag{A-5}$$

The wave functions of b and q_k are given by

$$\bar{u}_k\sigma_{\mu\nu}[(1+\gamma_5)/2]u_b = \frac{i}{2}\begin{pmatrix} \psi_{kL} \\ \psi_{kR} \end{pmatrix}^\dagger \times \begin{pmatrix} \tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu & 0 \\ 0 & \sigma_\mu\tilde{\sigma}_\nu - \sigma_\nu\tilde{\sigma}_\mu \end{pmatrix}\begin{pmatrix} 0 \\ \psi_{bR} \end{pmatrix} = (i/2)\psi_{kL}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)\psi_{bR}. \tag{A-6}$$

Putting in the penguin amplitude

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a) \times[\bar{u}_{kL}\frac{q^\nu}{q^2}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)u_{bR}] \times[\bar{u}_i(\tilde{\sigma}^\mu + \sigma^\mu)v_{\bar{j}}]. \tag{A-7}$$

Putting $q^\nu = (p_b - p_k)^\nu$ in the above equation,

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[(\bar{u}_{kL}\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)u_{bR}) -(\bar{u}_{kL}(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu u_{bR})] \times[\bar{u}_i\tilde{\sigma}^\mu v_{\bar{j}} + \bar{u}_i\sigma^\mu v_{\bar{j}}]. \tag{A-8}$$

or

$$M^{dip} =$$
$$-\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[\langle k_L|\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)|b_R\rangle$$
$$-\langle k_L|(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu|b_R\rangle]$$
$$\times[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A-9)}$$

We known that,

$$\tilde{\sigma}_\mu p_b^\mu|b_L\rangle = m_b|b_R\rangle,\ \tilde{\sigma}_\mu p_k^\mu|k_L\rangle = m_k|k_R\rangle,$$
$$\sigma_\nu p_b^\nu|b_R\rangle = m_b|b_L\rangle,\ \sigma_\nu p_k^\nu|k_R\rangle = m_k|k_L\rangle,$$
$$\tilde{\sigma}_\mu(\sigma_\nu p_k^\nu) = p_{k\mu}, \quad (\tilde{\sigma}_\mu p_b^\nu)\sigma_\mu = p_{b\mu}. \quad \text{(A-10)}$$

and

$$(p_b + p_k)_\mu = (2p_b - p_i - p_j)_\mu. \quad \text{(A-11)}$$

Also according to conservation of current

$$(p_i + p_j)_\mu[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]$$
$$= m_i[\langle i_R|j_L\rangle + \langle i_L|j_R\rangle]$$
$$-m_j[\langle i_L|j_R\rangle + \langle i_R|j_L\rangle] = 0, \quad \text{(A-12)}$$

since in the penguin decays is $m_i = m_j$ and $|j\rangle$ is the antiparticle, so

$$\tilde{\sigma}_\mu p_{j\mu}|j_L\rangle = -m_j|j_R\rangle. \quad \text{(A-13)}$$

Consequently, the magnetic dipole term of penguin amplitude becomes to

$$M^{dip} =$$
$$-\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)$$
$$\times\frac{1}{q^2}[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle + m_k\langle k_R|\sigma_\mu|b_R\rangle$$
$$-(p_b + p_k)_\mu\langle k_L|b_R\rangle] \times[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A-14)}$$

Term $m_k\langle k_R|\sigma_\mu|b_R\rangle$ was neglected because $m_k << m_b$, so

$$\begin{aligned}M^{dip} &= (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &+ m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle \\ &-(p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &-(p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle].\end{aligned} \quad \text{(A-15)}$$

Using (A-12) for the second part of the equation, we can write

$$\begin{aligned}M^{dip} &= (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &+ m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle \\ &-2p_{b\mu}\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &-2p_{b\mu}\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle].\end{aligned} \quad \text{(A-16)}$$

Here b meson is at the rest ($p_{b\mu} = (m_b, \vec{0})$) and

$$(T^aT^a) = 4/3,$$
$$d_8 = -\frac{g_s^2}{8\pi^2}F_2^R(0) = -\frac{g_s^2}{8\pi^2}m_b\frac{g_2^2}{8M_W^2}\sum_i V_{ik}^*V_{ib}f_2(x_i),$$
$$= -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^*V_{ib}f_2(x_i). \quad \text{(A-17)}$$

So, the magnetic dipole of penguin amplitude is given by

$$\begin{aligned}M^{dip} &= (4/3)d_8(m_b/q^2)[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &+\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle \\ &-2\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle].\end{aligned} \quad \text{(A-18)}$$

The b quark is at the rest and to have spin projection –1/2 along angle θ_b, thus the spin projection of b quark of +1/2 is along $\theta_b - \pi$ ($\theta_b \to \theta_b - \pi$),

$$\text{b spin}(-1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2})\begin{pmatrix}-\sin(\theta_b/2)\\ \cos(\theta_b/2)\end{pmatrix},$$
$$\text{b spin}(+1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2})\begin{pmatrix}\cos(\theta_b/2)\\ \sin(\theta_b/2)\end{pmatrix}. \quad \text{(A-19)}$$

Putting the factor of $(1/\sqrt{2})$ in the M^{dip} and neglecting the terms $\langle k_L|b_R\rangle$, thus the amplitude of magnetic dipole becomes

$$\begin{aligned}M^{dip} &= A_8d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle \\ &+\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle],\end{aligned} \quad \text{(A-20)}$$

here

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2). \quad \text{(A-21)}$$

Terms $(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu)$ and $(\tilde{\sigma}_\mu)(\sigma^\mu)_{LR}$ for spin +1/2 and –1/2 are obtained by the matrix elements of $L-L$ handed and $L-R$ handed for the b quark

$$\begin{aligned}&\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\langle -_k|_L\,\tilde{\sigma}_\mu|-_j\rangle_L = \\ &\sin((\theta_k - \theta_j - \theta_i)/2) + \sin((\theta_k + \theta_j - \theta_i)/2), \\ &\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\langle -_k|_L\,\tilde{\sigma}_\mu|-_j\rangle_L = \\ &\cos((\theta_k - \theta_j - \theta_i)/2) - \cos((\theta_k + \theta_j - \theta_i)/2),\end{aligned} \quad \text{(A-22)}$$

when dealing with penguin amplitudes, the following matrix elements are needed

$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\langle -_k|_R\,\sigma_\mu|-_j\rangle_R =$$
$$\sin((\theta_i - \theta_k - \theta_j)/2) - \sin((\theta_i + \theta_k - \theta_j)/2),$$
$$\langle -_i|_L\,\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\langle -_k|_R\,\sigma_\mu|-_j\rangle_R =$$
$$\cos((\theta_i - \theta_k - \theta_j)/2) + \cos((\theta_i + \theta_k - \theta_j)/2). \quad \text{(A-23)}$$

The first term of (A-20) for b spin project –1/2, according to Fierz transformation,

$$\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle = -\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle. \quad \text{(A-24)}$$

is given by

$$M^{dip}_{1(-1/2)} \equiv - A_8 d_8 \langle k_L | \tilde{\sigma}^{\mu} | b_L \rangle \langle i_L | \tilde{\sigma}_{\mu} | j_L \rangle = A_8 d_8 [-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2) - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)]. \quad \text{(A-25)}$$

And the first term for b spin project +1/2 is given by

$$M^{dip}_{1(+1/2)} \equiv - A_8 d_8 \langle k_L | \tilde{\sigma}^{\mu} | b_L \rangle \langle i_L | \tilde{\sigma}_{\mu} | j_L \rangle = A_8 d_8 [-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2) + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)]. \quad \text{(A-26)}$$

Also the second term of (A-20) for b spin project –1/2 is given by

$$M^{dip}_{2(-1/2)} \equiv - A_8 d_8 \langle k_L | \tilde{\sigma}^{\mu} | b_L \rangle \langle i_R | \sigma_{\mu} | j_R \rangle = A_8 d_8 [-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2) - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)]. \quad \text{(A-27)}$$

In addition, the second term for b spin project +1/2 is given by

$$M^{dip}_{2(+1/2)} \equiv - A_8 d_8 \langle k_L | \tilde{\sigma}^{\mu} | b_L \rangle \langle i_R | \sigma_{\mu} | j_R \rangle = A_8 d_8 [-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2) + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)]. \quad \text{(A-28)}$$

So the penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by

$$\begin{aligned} M^{dip}_{(+1/2)} = A_8 d_8 \{ & [-\sin((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ & + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ & + [-\sin((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ & + \sin((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \}, \end{aligned} \quad \text{(A-29)}$$

$$\begin{aligned} M^{dip}_{(-1/2)} = A_8 d_8 \{ & [-\cos((\theta_b - \theta_i - \theta_k + \theta_j)/2) \\ & - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \\ & + [-\cos((\theta_b + \theta_i - \theta_k - \theta_j)/2) \\ & - \cos((\theta_b - \theta_i + \theta_k - \theta_j)/2)] \}. \end{aligned} \quad \text{(A-30)}$$

Appendix B

Decay rate of the effective Hamiltonian

The effective $\Delta B = 1$ Hamiltonian at scale $\mu = \mathrm{O}(m_b)$ for tree plus penguin and including the electroweak penguin and the magnetic dipole term is

$$\begin{aligned} H^{\Delta B=1}_{eff} = 2\sqrt{2} G_F \{ & [d_{1c}(\mu) Q^c_1(\mu) + d_{2c}(\mu) Q^c_2(\mu)] \\ & + [d_{1u}(\mu) Q^u_1(\mu) + d_{2u}(\mu) Q^u_2(\mu)] \\ & - [\sum_{i=3}^{6,8} d_i(\mu) Q_i(\mu) + \sum_{i=7}^{10} d'_i(\mu) Q'_i(\mu)] \}, \end{aligned} \quad \text{(B-1)}$$

here $d_{1,\ldots,6}, d'_{7,\ldots,10}$ are defined by eq. (26), $d_{1,2c,u} = d_{1,2}(i = j = c,u)$ and index k refer to d or s. Using (A-22) and (A-23) one can obtain the matrix elements of the effective Hamiltonian operators. In the first step, the tree plus penguin operators $(Q_1,\ldots,Q_6)$ are chosen. All of the terms Q_1, Q_2, Q_3, Q_4 have a form L-L handed but terms Q_1, Q_2 have a form $\langle i|_L \tilde{\sigma}^{\mu} |b\rangle_L \langle k|_L \tilde{\sigma}_{\mu} | j\rangle_L$ and terms Q_3, Q_4 have a form $\langle k|_L \tilde{\sigma}^{\mu} |b\rangle_L \langle i|_L \tilde{\sigma}_{\mu} | j\rangle_L$. Consider i,k and j momenta in XZ plane, so according to (A-22), for the Q_1, Q_2 and, we can write for b spin projection 1/2 and –1/2. Also terms Q_1, Q_2 and Q_3, Q_4 differ only by a minus, because

$$\sin((\theta_i + \theta_j - \theta_k)/2) = -\sin((\theta_k - \theta_j - \theta_i)/2),$$

$$\sin((\theta_i - \theta_j - \theta_k)/2) = -\sin((\theta_k + \theta_j - \theta_i)/2). \quad \text{(B-2)}$$

Terms Q_5, Q_6 are of the form L-R handed, $\langle i|_L \tilde{\sigma}^{\mu} |b\rangle_L \langle k|_R \sigma_{\mu} | j\rangle_R$. The main forms of the terms Q_5, Q_6 are $\langle k|_L \tilde{\sigma}^{\mu} |b\rangle_L \langle i|_R \sigma_{\mu} | j\rangle_R$. These terms can be written according to (A-23). So, the matrix element for b quark spin project 1/2 is given by

$$\begin{aligned} M_{eff} = 2\sqrt{2} G_F \{ & (A_1 + A_2)[\sin((\theta_k - \theta_j - \theta_i)/2) \\ & + \sin((\theta_k + \theta_j - \theta_i/2)] \\ & - A_3[\sin((\theta_k - \theta_i - \theta_j)/2) \\ & - \sin((\theta_k + \theta_i - \theta_j)/2)] \}. \end{aligned} \quad \text{(B-3)}$$

here A_1, A_2 and A_3 are combination of Wilson coefficients and colour factors. The forms of $(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})_{LL}$ and $(\tilde{\sigma}^{\mu})(\sigma_{\mu})_{LR}$ are according to (A-22) and (A-23) respectively. So the square of spin average term $Q_1,\ldots,Q_6$ is given by

$$\begin{aligned} & [(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})_{LL} + (\tilde{\sigma}^{\mu})(\sigma_{\mu})_{LR}]^2_{sp-av} \\ & = \alpha_1 (1/16)(1+v_i)(1+v_k)(1+v_j)[1 - \cos(\theta_k - \theta_i)] \\ & + \alpha_2 (1/16)(1-v_i)(1+v_k)(1-v_j)[1 + \cos(\theta_k - \theta_j)] \\ & + \alpha_3 (1/16)\sqrt{1-v_i^2}(1+v_k)\sqrt{1-v_j^2}[1 + \cos(\theta_j - \theta_i) \\ & - \cos(\theta_k - \theta_j) - \cos(\theta_k - \theta_i)]. \end{aligned} \quad \text{(B-4)}$$

Now, one must obtain all of the helicity states for $Q_1,\ldots,Q_6$ and then sum them all. Adding eight terms of helicity states, on finds

$$\begin{aligned} & [(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})_{LL} + (\tilde{\sigma}^{\mu})(\sigma_{\mu})_{LR}]^2_{sp-av} = \\ & (\alpha_1/2)[1 - v_i v_k \cos(\theta_k - \theta_i)] \\ & + (\alpha_2/2)[1 + v_k v_j \cos(\theta_j - \theta_k)] \\ & + (\alpha_3/2)\sqrt{1-v_i^2}\sqrt{1-v_j^2}. \end{aligned} \quad \text{(B-5)}$$

After adding all colour combinations α_1, α_2 and α_3 gives

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2,$$

$$\alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2),$$

$$\begin{aligned}\alpha_3 = \mathrm{Re}\{&(3d_1 + d_2 + d_3 + 3d_4)d_6^* \\ &+ (d_1 + 3d_2 + 3d_3 + d_4)d_5^*\},\end{aligned} \quad \text{(B-6)}$$

here $d_1,...,d_6$ are defined by eq. (26). The energy conservation gives

$$\begin{aligned}\cos(\theta_k - \theta_i) = [(M_l - E_i - E_k)^2 \\ - (m_j^2 + p_i^2 + p_k^2)] / 2 p_i p_k,\end{aligned}$$

$$\begin{aligned}\cos(\theta_j - \theta_k) = [(M_l - E_k - E_j)^2 \\ - (m_i^2 + p_k^2 + p_j^2)] / 2 p_k p_j.\end{aligned} \quad \text{(B-7)}$$

The angle between the particle velocities must be physical, $-1 \le \cos(\theta_k - \theta_i) \le 1$ and $-1 \le \cos(\theta_j - \theta_k) \le 1$. So we should take the variable p_i and p_k, or x and y as,

$$p_i = xM_l / 2, \quad p_k = yM_l / 2 \quad \text{(B-8)}$$

Then p_j is given by energy conservation $E_j = M_b - E_i - E_k = \sqrt{m_j^2 + p_j^2}$. Also $\cos(\theta_i - \theta_k) = (p_j^2 - p_i^2 - p_k^2) / 2 p_i p_k$, and so on. Momentum conservation gives

$$p_k \cos(\theta_i - \theta_k) + p_j \cos(\theta_j - \theta_i) = -p_i, \quad \text{(B-9)}$$

and so on. Also

$$p_k \sin(\theta_i - \theta_k) = \pm p_j \sin(\theta_j - \theta_i). \quad \text{(B-10)}$$

and so on. The partial decay rate, b spin averaged and summed over final spin states, has overall spherical symmetry. Apart from its overall orientation, a final state is specified by only two parameters, say $p_i = |\boldsymbol{p_i}|$ and $p_k = |\boldsymbol{p_k}|$. The partial decay rate in the b rest frame is

$$\begin{aligned}d^2\Gamma_{Q_1,...,Q_6} / dp_i dp_k = (G_F^2 / \pi^3) p_i p_k E_{\bar{j}} \{\alpha_1 (p_i . p_k / E_i E_k) \\ + \alpha_2 (p_i . p_{\bar{j}} / E_i E_{\bar{j}}) + \alpha_3 (m_k m_{\bar{j}} / E_k E_{\bar{j}})\},\end{aligned} \quad \text{(B-11)}$$

here

$$p_i . p_k = (M_b^2 + m_{\bar{j}}^2 - m_i^2 - m_k^2 - 2M_b E_{\bar{j}}) / 2,$$

$$p_i . p_{\bar{j}} = (M_b^2 + m_k^2 - m_i^2 - m_{\bar{j}}^2 - 2M_b E_k) / 2. \quad \text{(B-12)}$$

After the change of variable to x and y, the decay rate is given by

$$d^2\Gamma_{Q_1,...,Q_6} / dxdy = \Gamma_{0b} I_{ps}^{EH}, \quad \text{(B-13)}$$

$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3, \quad \text{(B-14)}$$

where

$$I_{ps}^1 = 6xy.f_{ab}.(1 - h_{abc}),$$

$$I_{ps}^2 = 6xy.f_{bc}.(1 + h_{bca}),$$

$$I_{ps}^3 = 6xy.f_{ac}.h_{xa}.h_{yc}. \quad \text{(B-15)}$$

Here

$$f_{ab} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + b^2},$$

$$h_{abc} = \frac{(f_{ab})^2 - (c^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + b^2}},$$

$$f_{bc} = 2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2},$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2 + x^2 + y^2)}{2\sqrt{x^2 + b^2}\sqrt{y^2 + c^2}},$$

$$f_{ac} = 2 - \sqrt{x^2 + a^2} - \sqrt{y^2 + c^2},$$

$$h_{acb} = \frac{(f_{ac})^2 - (b^2 + x^2 + y^2)}{2\sqrt{x^2 + a^2}\sqrt{y^2 + c^2}}.$$

$$h_{xa} = [1 - (x^2 / (x^2 + a^2))]^{1/2},$$

$$h_{yc} = [1 - (y^2 / (y^2 + c^2))]^{1/2}, \quad \text{(B-16)}$$

where a, b and c are:

$$a = 2m_i / M_b, \; b = 2m_k / M_b, \; c = 2m_j / M_b. \quad \text{(B-17)}$$

Appendix C

Effective Hamiltonian of magnetic dipole decay rate

We calculate the decay rates of $b \to q_i q_k \bar{q}_j$ according to effective Hamiltonian ($Q_1,...,Q_6$), including magnetic dipole (Q_8) terms. The amplitude of Effective Hamiltonian for operators Q_1, Q_2, Q_3, Q_4 is given by

$$\begin{aligned}M1_{(1/2)}^{L-L} = A(d_1 + d_2 + d_3 + d_4)[\sin((\theta_k - \theta_j - \theta_i)/2) \\ + \sin((\theta_k + \theta_j - \theta_i)/2)],\end{aligned}$$

$$\begin{aligned}M2_{(-1/2)}^{L-L} = A(d_1 + d_2 + d_3 + d_4)[\cos((\theta_k - \theta_j - \theta_i)/2) \\ - \cos((\theta_k + \theta_j - \theta_i)/2)],\end{aligned}$$

(C-1)

and for operators Q_5, Q_6 is as well

$$\begin{aligned}M3_{(1/2)}^{L-R} = A'(d_5 + d_6)[\sin((\theta_k - \theta_i - \theta_j)/2) \\ - \sin((\theta_k + \theta_i - \theta_j)/2)],\end{aligned}$$

$$\begin{aligned}M4_{(-1/2)}^{L-R} = A'(d_5 + d_6)[\cos((\theta_k - \theta_i - \theta_j)/2) \\ + \cos((\theta_k + \theta_i - \theta_j)/2)],\end{aligned} \quad \text{(C-2)}$$

where $d_1,...,d_6, d_8$ and A_8 are defined by eq. (26) and (A-21) respectively and

$$A = \sqrt{(1 + v_i)}\sqrt{(1 + v_k)}\sqrt{(1 + v_{\bar{j}})},$$

$$A' = \sqrt{(1 - v_i)}\sqrt{(1 + v_k)}\sqrt{(1 - v_{\bar{j}})}, \quad \text{(C-3)}$$

Also the amplitude of magnetic dipole according to (A-20) is given by

$$\begin{aligned}M^{dip} = A_8 d_8 [&\langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_L | \tilde{\sigma}^\mu | j_L \rangle \\ &+ \langle k_L | \tilde{\sigma}_\mu | b_L \rangle \langle i_R | \sigma^\mu | j_R \rangle],\end{aligned} \quad \text{(C-4)}$$

For various b spin project 1/2 and –1/2 the first term of (C-4) is given by

$$M5^{dip,L-L}_{(1/2)} = AA_8 d_8[\sin((\theta_i - \theta_k + \theta_j)/2) + \sin((\theta_i + \theta_k - \theta_j)/2)],$$

$$M6^{dip,L-L}_{(-1/2)} = AA_8 d_8[-\cos((\theta_i - \theta_k + \theta_j)/2) - \cos((\theta_i + \theta_k - \theta_j)/2)], \quad \text{(C-5)}$$

and the second term of (C-4) is given by

$$M7^{dip,L-R}_{(1/2)} = A'A_8 d_8[\sin((\theta_i - \theta_k + \theta_j)/2) - \sin((\theta_i - \theta_k - \theta_j)/2)],$$

$$M8^{dip,L-R}_{(-1/2)} = A'A_8 d_8[-\cos((\theta_i - \theta_k + \theta_j)/2) - \cos((\theta_i - \theta_k - \theta_j)/2)]. \quad \text{(C-6)}$$

Now the amplitude of $Q_1,...,Q_6$ and Q_8 (magnetic dipole term) should be added for b spine project 1/2 and –1/2, so

$$M^{tot}_{(1/2)} = M1^{L-L}_{(1/2)} + M3^{L-R}_{(1/2)} + M5^{dip,L-L}_{(1/2)} + M7^{dip,L-R}_{(1/2)},$$

$$M^{tot}_{(-1/2)} = M2^{L-L}_{(-1/2)} + M4^{L-R}_{(-1/2)} + M6^{dip,L-L}_{(-1/2)} + M8^{dip,L-R}_{(-1/2)}, \quad \text{(C-7)}$$

or

$$M^{tot}_{(-1/2)} = e_1 \cos((\theta_k - \theta_i - \theta_j)/2) - e_2 \cos((\theta_k - \theta_i + \theta_j)/2) + e_3 \cos((\theta_k + \theta_i - \theta_j)/2),$$

$$M^{tot}_{(1/2)} = e_1 \sin((\theta_k - \theta_i - \theta_j)/2) + e_2 \sin((\theta_k - \theta_i + \theta_j)/2) - e_3 \sin((\theta_k + \theta_i - \theta_j)/2), \quad \text{(C-8)}$$

here

$$e_1 = A[(d_1 + d_2 + d_3 + d_4) + A_8 d_8] + A'[(d_5 + d_6) + A_8 d_8],$$

$$e_2 = A(d_1 + d_2 + d_3 + d_4) - A' \ A_8 d_8,$$

$$e_3 = AA_8 d_8 + A'(d_5 + d_6). \quad \text{(C-9)}$$

The spin average of b spin project of 1/2 and –1/2 is given by

$$\left|M^{tot}\right|^2_{spin-ave} = \frac{1}{2}[\left|M^{tot}_{(1/2)}\right|^2 + \left|M^{tot}_{(-1/2)}\right|^2]$$
$$= \frac{1}{2}[e_1^2 + e_2^2 + e_3^2 - 2e_1 e_2 \cos(\theta_k - \theta_i) + 2e_1 e_3 \cos(\theta_j - \theta_k) - 2e_2 e_3 \cos(\theta_j - \theta_i)]. \quad \text{(C-10)}$$

After adding all color factors, it gives

$$e_1 = A[h_1 + h_2] + A' \ [h_2 + h_3], \ e_2 = Ah_1 - A' \ h_2,$$

$$e_3 = Ah_2 + A' \ h_3, \quad \text{(C-11)}$$

where

$$h_1 = \sqrt{\left|d_1 + d_2 + d_3 + d_4\right|^2 + 2\left|d_1 + d_4\right|^2 + 2\left|d_2 + d_3\right|^2},$$

$$h_2 = A_8 d_8, \ h_3 = \sqrt{\left|d_5 + d_6\right|^2 + 2\left|d_5\right|^2 + 2\left|d_6\right|^2}, \quad \text{(C-12)}$$

The first term of (C-10) is given by

1)

$$e_1^2 + e_2^2 + e_3^2 = A^2(2h_1^2 + 2h_2^2 + 2h_1 h_2) + A'(2h_2^2 + 2h_3^2 + 2h_2 h_3) + 2AA'(h_2^2 + h_1 h_3 + 2h_2 h_3), \quad \text{(C-13)}$$

Adding eight terms of helicity states, so

$$e_1^2 + e_2^2 + e_3^2 = 8(g_1 + g_2 + 2\sqrt{1 - v_i^2}\sqrt{1 - v_j^2} \ g_3), \quad \text{(C-14)}$$

The second term of (C-10) is given by

2)

$$2e_1 e_2 \cos(\theta_k - \theta_i) = 2\{A^2(h_1^2 + h_1 h_2) - A'^2(h_2^2 + h_2 h_3) + AA'(h_1 h_3 - h_2^2)\}\cos(\theta_k - \theta_i), \quad \text{(C-15)}$$

Adding eight terms of helicity states, so

$$2e_1 e_2 \cos(\theta_k - \theta_i) = 2.8.v_k v_i (g_4 - g_5)\cos(\theta_k - \theta_i), \quad \text{(C-16)}$$

The third term of (C-10) is given by

3)

$$2e_1 e_3 \cos(\theta_j - \theta_k) = 2\{A^2(h_2^2 + h_1 h_2) + A'^2(h_3^2 + h_2 h_3) + AA'(h_1 h_3 + 2h_2 h_3 + h_2^2)\}\cos(\theta_j - \theta_k). \quad \text{(C-17)}$$

Adding eight terms of helicity states, so

$$2e_1 e_3 \cos(\theta_j - \theta_k) = 2.8.v_j v_k (g_6 + g_7)\cos(\theta_j - \theta_k), \quad \text{(C-18)}$$

the fourth term of (C-10) is given by

4)

$$2e_2 e_3 \cos(\theta_j - \theta_i) = 2\{A^2(h_1 h_2) - A'^2(h_2 h_3) + AA'(h_1 h_3 - h_2^2)\}\cos(\theta_j - \theta_i), \quad \text{(C-19)}$$

adding eight terms of helicity states, so

$$2e_2 e_3 \cos(\theta_j - \theta_i) = 2.8.v_j v_i (g_8 - g_9)\cos(\theta_j - \theta_i), \quad \text{(C-20)}$$

here

$$g_1 = 2h_1^2 + 2h_2^2 + 2h_1 h_2, \qquad g_2 = 2h_2^2 + 2h_3^2 + 2h_2 h_3,$$
$$g_3 = h_2^2 - h_1 h_2 + h_1 h_3 + 3h_2 h_3, \quad g_4 = h_1^2 + h_1 h_2,$$
$$g_5 = h_2^2 + h_2 h_3, \quad g_6 = h_2^2 + h_1 h_2, \qquad g_7 = h_3^2 + h_2 h_3,$$
$$g_8 = h_1 h_2, \quad g_9 = h_2 h_3, \quad \text{(C-21)}$$

The total amplitude of (C-10) is given by

$$\left|M^{tot}\right|^2_{spin-ave} = \frac{1}{4}[g_1 + g_2 - 2\sqrt{1 - v_i^2}\sqrt{1 - v_j^2} g_3 - 2.v_k v_i (g_4 - g_5)\cos(\theta_k - \theta_i) + 2.v_j v_k (g_6 + g_7)\cos(\theta_j - \theta_k) - 2.v_j v_i (g_8 - g_9)\cos(\theta_j - \theta_i)], \quad \text{(C-22)}$$

or

$$\left|M^{tot}\right|^2_{spin-ave} = \frac{1}{4}[g_1 - 2.v_k v_i (g_4 - g_5)\cos(\theta_k - \theta_i) + g_2 + 2.v_j v_k (g_6 + g_7)\cos(\theta_j - \theta_k) - 2\sqrt{1 - v_i^2}\sqrt{1 - v_j^2} g_3 - 2.v_j v_i (g_8 - g_9)\cos(\theta_j - \theta_i)], \quad \text{(C-23)}$$

Also one can obtain the amplitude of tree-level and Effective Hamiltonian $(Q_1,...,Q_6)$. The amplitude of tree-level ($d_2 = d_3 = d_4 = d_5 = d_6 = d_8 = 0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,TL} = \frac{1}{4}[2h_1^2 - 2.h_1^2 v_i v_k \cos(\theta_k - \theta_i) + 0 + 0]$$

$$= 3d_1^2 . \frac{1}{2}(1 - v_i v_k \cos(\theta_k - \theta_i)) \,, \quad \text{(C-24)}$$

and the amplitude of effective Hamiltonian ($d_8 = 0$) is given by

$$\left|M^{tot}\right|^2_{spin-ave,EH} = (h_1^2/2)[1 - v_k v_i \cos(\theta_k - \theta_i)] + (h_3^2/2)[1 + v_j v_k \cos(\theta_j - \theta_k)] - (\sqrt{1-v_i^2}\sqrt{1-v_j^2}/4)(2h_1 h_3) \,. \quad \text{(C-25)}$$

After integration in the phase space and changing the variables x and y,

$$x = 2p_i / m_b \,, \; y = 2p_k / m_b \,, \quad \text{(C-26)}$$

the differential of decay rate of effective Hamiltonian plus magnetic dipole (C-23) is given by

$$\frac{d^2\Gamma}{dxdy} = \frac{G_F^2 M_b^2}{192\pi^3}\frac{1}{2}(I_1 + I_2 + I_3) \,, \quad \text{(C-27)}$$

where

$$I_1 = 6xyf_{ab}\,[g_1 - 2(g_4 - g_5)h_{abc}] \,,$$
$$I_2 = 6xyf_{bc}\,[g_2 + 2(g_6 + g_7)h_{bca}] \,,$$
$$I_3 = 6xyf_{ac}\,[-2g_3 h_{xa} h_{yc} - 2(g_8 - g_9)h_{acb}] \,, \quad \text{(C-28)}$$

and $f_{ab}, f_{bc}, f_{ac}, h_{abc}, h_{bca}, h_{acb}, h_{xa}$, h_{yc} defined by (B-16).

References

1. Z Xiao, C S Li, K T Chao, *Phys. Lett.* B **473** (2000) 148-156.
2. M Beneke, J Rohrer and D Yang, *Phys. Rev. Lett.* **96** (2006) 141801.
3. S A Abel, W N Cotingham and I B Wittingham, *Phys. Rev.* D **58** (1998) 073006.
4. A J Buras, R Fleischer, S Recksiegel, F Schwab, the *Proceedings of EPS-HEP* (2005).
5. C D Lu, Y L Shen, W Wang, *Chin. Phys. Lett.* **23** (2006) 2684-2687.
6. M Beneke, S Jager, *Nucl. Phys.* B **768** (2007) 51-84.
7. W N Cottingham, H Mehrban, I B Wittingham, *Phys. Rev.* D **60** (1999) 114029.
8. H N Li, *Czech. J. Phys.* **53** (2003) 657-666, *Prog. Part. Nucl. Phys.* **51** (2003) 85-171.
9. Z Xiao, W Li, L Guo, G Lu, *Eur. Phys. J. C* **18** (2001) 681-709.
10. A J Buras, P Gambino, U Haisch, *Nucl. Phys.* B **570** (2000) 117-154.
11. S Actis, G Passarino, *Equations and their Solutions*, hep-ph/0612124.
12. A J Buras, P Gambino, U Haisch, *Nucl. Phys.* B **570** (2000) 117-154.
13. J B Mendy, J Govaerts, To be published in the Proceedings of the Second International Workshop on Contemporary Problems in Mathematical Physics, Cotonou, Republic of Benin.
14. N G Deshpande, X G He, S Pakvasa, *Phys. Lett.* B **326** (1994) 307-311.
15. A J Buras, *Nucl. Instrum. Meth.* A **368** (1995) 1-20.
16. A Ali, C Greub, *Phys. Rev.* D **57** (1998) 2996-3016.
17. G Burdman, Ian. Shipsey, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 431-499.
18. A Datta, D London, *JHEP* **0404** (2004) 072.
19. Particle Data Group, *European Physical J.* C **3** (2007) 1.
20. H Y Cheng, hep-ph/9807323.
21. N G Deshpande, B Dutta, S Oh, *Phys. Rev.* D **57** (1998) 5723-5731.
22. H Y Cheng, B Tseng, *Phys. Rev.* D **58** (1998) 094005.
23. Y H Chen, H Y Cheng, B Tseng, *Phys. Rev.* D **59** (1999) 074003.