Bulk viscous string cosmological models in Saez - Ballester theory of gravitation

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Abstract:

Spatially homogeneous Bianchi type-II, VIII & IX anisotropic as well as isotropic cosmological models is obtained in a scalar tensor theory of gravitation proposed by Saez and Ballester (1986) when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. All the models obtained and presented here are expanding, non-rotating and accelerating. Also some important features of the models, thus obtained, have been discussed.

Keywords: Bianchi type - II, VIII & IX metrics, Bulk viscosity, Cosmic strings, Saez – Ballester theory.

1. Introduction:

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of the large scale behavior of the Universe. In search of a realistic picture of the early Universe such models have been widely studied within a framework of General Relativity. In order to study the evolution of the universe, many authors constructed cosmological models containing viscous fluid. Cosmological models with bulk-viscosity are important since bulk-viscosity has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase. The role of bulk viscosity in the cosmic evolution, especially as its early stages, seems to be significant. The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our universe. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. The presence of viscosity in the fluid gives many interesting features in the dynamics of homogeneous cosmological models. The possibility of bulk viscosity leading to inflationary like solutions in general relativistic FRW models has been discussed by several authors. Barrow (1986), Padmanabhan and Chitre (1987), Pavon et al. (1991), Martens, (1995), Lima et al. (1993)]. Wang (2004, 2005, 2006), Bali and Dave (2002), Bali and Pradhan (2007), Tripathy et al. (2009), Tripathy et al. (2010) and Rao et al. (2011) have studied various Bianchi type cosmological models in the presence of cosmic strings and bulk viscosity. Rao and Sireesha (2012 b) have discussed axially symmetric string cosmological model with bulk viscosity in self-creation theory of gravitation.

The study of string theory has received considerable attention in cosmology. String cosmological models are attracting more and more attention of research workers since cosmic strings are important in the early stages of evaluation of the universe before the practical creation. Spontaneous symmetry breaking in elementary practical physics has given rise to topological defects known as cosmic strings. The gravitational effects of such objects are of practical interest since they are considered as possible seeds for galaxy formation and gravitational lenses. Rao et al.(2008c) have obtained Bianchi type-II,VIII & IX string cosmological models in Brans-Dicke theory of gravitation. Rao et al. (2009) have studied axially symmetric string cosmological models in Brans-Dicke theory of gravitation.

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.

We assume the lagrangian

$$L = R - W\phi^n \phi_{,\gamma}\phi^{,\gamma} \tag{1.1}$$

R being the scalar curvature, ϕ a dimensionless scalar field, W and n arbitrary dimensionless constants and $\phi^{,\gamma}$ the contraction $\phi_{,\alpha} g^{\alpha\gamma}$ (the partial derivatives are denoted by a comma and the covariant derivatives by a semicolon in the usual way).

For a scalar field having the dimensions of G^{-1} the lagrangian (1.1) is not physically admissible because the two terms of the right-hand side of eq. (1.1) have different dimensions. However it is a suitable lagrangian in the case of a dimensionless scalar field.

From the above lagrangian we can build the action

$$I = \int_{\Sigma} (L + \chi L_m)(-g)^{1/2} dX^1 dX^2 dX^3 dX^4 \tag{1.2}$$

where L_m is the matter lagrangian, g is the determinant of the matrix g_{ij} , X^i are the coordinates, Σ is an arbitrary region of integration and $\chi = 8\pi$ (we use geometrized units).

By considering arbitrary independent variations of the metric and the scalar field vanishing at the boundary of Σ , the variational principle

$$\delta I = 0 \tag{1.3}$$

leads to the field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with c = 1, $8\pi G$ = 1) are

$$G_{ij} - \omega\phi^{m}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1.4}$$

and the scalar field ϕ satisfies the equation

$$2\phi^{m}\phi^{i}{}_{,i} + m\phi^{m-1}\phi_{,k}\phi^{,k} = 0 \tag{1.5}$$

where $G_{ij} = R_{ij} - \frac{1}{2}R g_{ij}$ is an Einstein tensor, R the scalar curvature, ω & m are constants and T_{ij} is the stress energy tensor of the matter.

Also the energy conservation equation is given by

$$T^{ij}{}_{,j} = 0 \tag{1.6}$$

The study of cosmological models in the framework of scalar–tensor theories has been the active area of research for the last few decades. In particular, Rao et al. (2008a, b) have investigated several aspects of the cosmological models in Saez-Ballester (1986) scalar-tensor theory. Rao et al. (2013a) have discussed Bianchi type - II, VIII & IX perfect fluid dark energy cosmological models in Saez - Ballester and general theory of gravitation. Also Rao et al. (2013b) have obtained perfect fluid dark energy cosmological models in this theory and Einstein’s theory of gravitation.

Bianchi type space- times play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi types II, VIII & IX universes are important because familiar solutions like FRW universe with positive curvature, the de Sitter universe, the Taub- Nut solutions etc. correspond

Bianchi type II, VIII & IX space- times. Shanthi & Rao (1991) studied Bianchi types VIII and IX models in Lyttleton-Bondi Universe. Also Rao and Sanyasi Raju, (1992) and Sanyasi Raju and Rao, (1992) have studied Bianchi types VIII & IX models in Zero mass scalar fields and self creation cosmology. Reddy et al. (1993) studied Bianchi types II, VIII & IX models in scale covariant theory of gravitation. Raj Bali and Dave (2001), Raj Bali and Yadav (2005) studied Bianchi type IX string as well as viscous fluid models in general relativity. Rahaman et al. (2003) have investigated Bianchi type- IX string cosmological model in a theory of gravitation formulated by Sen (1957) based on Lyra (1951) manifold. Naidu et al. (2012) have discussed LRS Bianchi type-II universe with cosmic strings and bulk viscosity in a scalar tensor theory of gravitation using negative constant deceleration parameter proposed by Bermann (1983). Rao et al. (2012) have studied Bianchi types II, VIII & IX string cosmological models with bulk viscosity in a theory of gravitation. Rao and Sireesha (2012 a) have discussed Bianchi types II, VIII & IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. Rao et al. (2013c) have obtained Bianchi type-I string cosmological models with bulk viscosity in Bimetric theory of gravitation. Rao and Neelima have investigated string cosmological models with bulk viscosity in Nordtvedt's general scalar tensor theory of gravitation. Rao et al. (2014a) have discussed Perfect fluid cosmological models in a modified theory of gravity. Recently, Rao et al. (2014b, c) have studied bulk viscous cosmological models, coupled with a scalar field in Einstein theory and f (R,T) theory of gravity respectively.

In this paper we will discuss spatially homogeneous Bianchi type-II, VIII &IX anisotropic as well as isotropic bulk viscous string cosmological models, without using negative constant deceleration parameter proposed by Bermann (1983), in Saez - Ballester (1986) theory of gravitation.

2. Metric and Energy Momentum Tensor:

We consider a spatially homogeneous Bianchi type-II, VIII & IX metrics of the form

$$ds^2 = dt^2 - R^2\left[d\theta^2 + f^2(\theta)d\varphi^2\right] - S^2\left[d\psi + h(\theta)d\varphi\right]^2 \qquad (2.1)$$

where (θ,φ,ψ) are the Eulerian angles, Rand S are functions of t only.

It represents

Bianchi type - II if $f(\theta)=1$ and $h(\theta)=\theta$

Bianchi type - VIII if $f(\theta)=Cosh\,\theta$ and $h(\theta)=Sinh\,\theta$

Bianchi type - IX if $f(\theta)=Sin\,\theta$ and $h(\theta)=Cos\,\theta$

The energy momentum tensor for a bulk viscous fluid containing one dimensional string is given by

$$T_{ij} = (\rho+\bar{p})u_i u_j - \bar{p}g_{ij} - \lambda x_i x_j \qquad (2.2)$$

and $\bar{p} = p - 3\xi H$, $p = \in_0 \rho \quad (0 \le \in_0 \le 1)$ (2.3)

where $\bar{p}$ is the total pressure which includes the proper pressure, ρ is the rest energy density of the system , λ is the tension in the string , $\xi(t)$ is the coefficient of bulk viscosity, $3\xi H$ is usually known as bulk viscous pressure, H is the Hubble parameter, u^i is the four velocity vector and x^i is a space-like vector which represents the anisotropic directions of the string.

Here u^i and x^i satisfy the equations

$$g_{ij}u^i u^j = -1,$$

$$g_{ij}x^i x^j = 1,$$

and $u^i x_i = 0.$ (2.4)

We assume that the string be lying along the z-axis. The one dimensional strings are assumed to be loaded with particles and the particle energy density is $\rho_p = \rho - \lambda$.

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = -\bar{p},\ T_3^3 = \lambda - \bar{p},\ T_4^4 = \rho \quad (2.5)$$

where $\rho, \lambda, \bar{p}$ and ϕ are functions of time 't' only.

3. Solutions of Field equations:

Now with the help of (2.2) to (2.5), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -\bar{p} \quad (3.1)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{3S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = \lambda - \bar{p} \quad (3.2)$$

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{S^2}{4R^4} + \frac{\omega}{2}\phi^m\dot{\phi}^2 = \rho \quad (3.3)$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) + \frac{m}{2\phi}\dot{\phi}^2 = 0 \quad (3.4)$$

$$\dot{\rho} + (\rho + \bar{p})(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}) - \lambda\frac{\dot{S}}{S} = 0 \ . \quad (3.5)$$

Here the over head dot denotes differentiation with respect to't'.

When $\delta = 0, -1 \,\&\, +1$ the field equations (3.1) to (3.5) correspond to the Bianchi types II, VIII & IX universes respectively.

Using the transformation $dt = R^2 S dT$, the above field equations (3.1) to (3.5) will reduce to

$$\left(\frac{R'}{R}\right)' + \left(\frac{S'}{S}\right)' - \left(\frac{R'}{R}\right)^2 - 2\left(\frac{R'S'}{RS}\right) + \frac{S^4}{4} - \frac{\omega}{2}\phi^m\phi'^2 = -\bar{p}R^4S^2 \qquad (3.6)$$

$$2\left(\frac{R'}{R}\right)' - \left(\frac{R'}{R}\right)^2 - 2\left(\frac{R'S'}{RS}\right) - \frac{3S^4}{4} + \delta R^2S^2 - \frac{\omega}{2}\phi^m\phi'^2 = (\lambda - \bar{p})R^4S^2 \qquad (3.7)$$

$$\left(\frac{R'}{R}\right)^2 + 2\left(\frac{R'S'}{RS}\right) - \frac{S^4}{4} + \delta R^2S^2 + \frac{\omega}{2}\phi^m\phi'^2 = \rho R^4S^2 \qquad (3.8)$$

$$\phi'' + \frac{m}{2\phi}\phi'^2 = 0 \qquad (3.9)$$

$$\rho' + (\rho + \bar{p})(2\frac{R'}{R} + \frac{S'}{S}) - \lambda\frac{S'}{S} = 0 \qquad (3.10)$$

Here the over head dash denotes differentiation with respect to ’T’.

The field equations (3.6) to (3.10) are only four independent equations with seven unknowns $R, S, \rho, \lambda, \bar{p}$ & ϕ, which are functions of ‘T’. Since these equations are non-linear in nature, in order to get a deterministic solution we take the following plausible physical conditions:

(1). The shear scalar σ is proportional to scalar expansion θ, so that we can take a linear relationship between the metric potentials R and S, i.e.,

$$R = S^n \qquad (3.11)$$

where n is an arbitrary constant.

(2). A more general relationship between the proper rest energy density ρ and string tension density λ is taken to be

$$\rho = r\lambda \qquad (3.12)$$

where r is an arbitrary constant which can take both positive and negative values. The negative value of r leads to the absence of strings in the universe and the positive value show the presence of one dimensional string in the cosmic fluid. The energy density of the particles attached to the strings is

$$\rho_p = \rho - \lambda = (r-1)\lambda \tag{3.13}$$

From equations (3.6) - (3.13), we get

$$\frac{S''}{S} r(n-1) - \frac{S'^2}{S^2}[r(n-1)+n(n+2)] - \frac{(4r-1)}{4} S^4 + \delta(r-1) S^{2n+2} - \frac{\omega}{2}\phi^m \phi'^2 = 0 \tag{3.14}$$

From equation (3.9), we get

$$\phi = \left[\left(\frac{m+2}{2}\right)(k_1 T + k_2)\right]^{\frac{2}{m+2}}, \ m \neq -2 \tag{3.15}$$

Bianchi type-II ($\delta = 0$) cosmological model:

If $\delta = 0$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 - \frac{\omega}{2} k_1^{\ 2} = 0 \tag{3.16}$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

With a suitable substitution, the equation (3.16) can be written as

$$S'^2 = W^2 S^6 - \gamma^2 S^2 \tag{3.17}$$

where $W^2 = \frac{(4r-1)}{4[2r(n-1)-n(n+2)]}$ & $\gamma^2 = \frac{\omega k_1^{\ 2}}{2n(n+2)}$.

From equation (3.17), we get

$$S^2 = \left(\frac{\gamma}{W}\right) Sec 2\gamma T \tag{3.18}$$

From equations (3.11) & (3.18), we get

$$R^2 = \left[\left(\frac{\gamma}{W}\right) Sec\, 2\gamma T\right]^n \tag{3.19}$$

From equations (3.8), (3.15), (3.18) & (3.19) we get the energy density

$$\rho = \frac{n(n+2)W^{(2n+1)}}{\gamma^{(2n-1)}} Tan^2 2\gamma T\, (Sec\, 2\gamma T)^{-(2n+1)} - \frac{1}{4}\left(\frac{\gamma}{W}\right)^{1-2n} (Sec 2\gamma T)^{1-2n}$$
$$+ \frac{\omega}{2} k_1{}^2 \left(\frac{\gamma}{W}\right)^{-(2n+1)} (Sec\, 2\gamma T)^{-(2n+1)}$$

(3.20)

From equations (3.6), (3.15), (3.18) & (3.19) we get the total pressure

$$\bar{p} = \left(\frac{\gamma}{W}\right)^{-(2n+1)} (Sec 2\gamma T)^{-(2n+1)} [n(n+2)\gamma^2\, Tan^2 2\gamma T + \frac{\omega}{2} k_1{}^2]$$
$$- (Sec 2\gamma T)^{1-2n} \left[2(n+1)\frac{W^{(2n+1)}}{\gamma^{(2n-1)}} + \frac{1}{4}\left(\frac{\gamma}{W}\right)^{1-2n}\right] \tag{3.21}$$

The proper pressure is given by

$$p = \in_0 \rho = \frac{n(n+2)W^{(2n+1)} \in_0}{\gamma^{(2n-1)}} Tan^2 2\gamma T\, (Sec\, 2\gamma T)^{-(2n+1)} - \frac{1}{4}\left(\frac{\gamma}{W}\right)^{1-2n} \in_0 (Sec 2\gamma T)^{1-2n}$$
$$+ \frac{\omega}{2} k_1{}^2 \in_0 \left(\frac{\gamma}{W}\right)^{-(2n+1)} (Sec\, 2\gamma T)^{-(2n+1)}$$

(3.22)

From equations (3.7), (3.15), (3.18), (3.19) & (3.21) we get the string tension density

$$\lambda = \left(\frac{\gamma}{W}\right)^{-(2n+1)} (Sec\, 2\gamma T)^{1-2n} \left[2(n-1)\gamma^2 - \left(\frac{\gamma}{W}\right)^2\right] \tag{3.23}$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = \left(\frac{\gamma}{W}\right)^{-(2n+1)} (Sec2\gamma T)^{1-2n}(r-1)\left[2(n-1)\gamma^2 - \left(\frac{\gamma}{W}\right)^2\right] \qquad (3.24)$$

The coefficient of bulk viscosity is given by

$$\xi = \left(\frac{\gamma}{W}\right)^{-(2n+1)} \frac{(1-\epsilon_0)}{(2n+1)\gamma}[n(n+2)\gamma^2 Tan^2 2\gamma T + \frac{\omega}{2}k_1{}^2](Sec\,2\gamma T)^{-(2n+1)} Cot\,2\gamma T$$
$$+\frac{1}{4}\left(\frac{\gamma}{W}\right)^{1-2n} \frac{(\epsilon_0 - 1)}{(2n+1)\gamma}(Sec2\gamma T)^{1-2n} Cot\,2\gamma T$$
$$-\left(\frac{\gamma}{W}\right)^{-(2n+1)} \frac{2(n+1)\gamma}{(2n+1)}(Sec2\gamma T)^{1-2n} Cot\,2\gamma T \qquad (3.25)$$

The generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{-(2n+1)\gamma}{3} Tan\,2\gamma T \qquad (3.26)$$

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{\gamma}{W}\right)Sec\,2\gamma T\right]^{\frac{2n+1}{2}} dT^2 - \left[\left(\frac{\gamma}{W}\right)Sec\,2\gamma T\right]^n (d\theta^2 + d\varphi^2)$$
$$-\left[\left(\frac{\gamma}{W}\right)Sec\,2\gamma T\right](d\psi + \theta d\varphi)^2 \qquad (3.27)$$

Thus (3.27) together with (3.15), (3.20), (3.21), (3.23) & (3.25) constitutes a Bianchi type-II string cosmological model with bulk viscosity in Saez - Ballester (1986) theory of gravitation. This model is entirely different from the model obtained by Naidu et al. (2012) since we have obtained it without using negative constant deceleration parameter proposed by Bermann (1983).

Bianchi type-VIII ($\delta = -1$) cosmological model:

If $\delta = -1$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 - (r-1) S^{2n+2} - \frac{\omega}{2} k_1^{\ 2} = 0 \tag{3.28}$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

For $n = -1$, with a suitable substitution from equation (3.28), we get

$$S'^2 = \gamma^2 S^2 - W^2 S^6 \tag{3.29}$$

where $\gamma^2 = \frac{2(r-1) + \omega k_1^{\ 2}}{2}$, $r < 0$ & $W^2 = \frac{1}{4}$.

From equation (3.29), we get

$$S^2 = \left(\frac{\gamma}{W}\right) Sech 2\gamma T \tag{3.30}$$

From equations (3.11) & (3.30), we get

$$R^{-2} = \left(\frac{\gamma}{W}\right) Sech 2\gamma T \tag{3.31}$$

From equations (3.8), (3.15), (3.30) & (3.31) we get

The energy density

$$\rho = -r \frac{\gamma}{W} Sech 2\gamma T \quad , \ r < 0 \tag{3.32}$$

From equations (3.6), (3.15), (3.30) & (3.31) we get

The total pressure

$$\bar{p} = (1-r) \frac{\gamma}{W} \ Sech 2\gamma T \tag{3.33}$$

The proper pressure is given by

$$p = \in_0 \rho = -r \in_0 \frac{\gamma}{W} Sech 2\gamma T \tag{3.34}$$

From equations (3.7), (3.15), (3.30) & (3.33) we get

The string tension density

$$\lambda = -\frac{\gamma}{W} Sech 2\gamma T \tag{3.35}$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = (1-r)\frac{\gamma}{W} Sech 2\gamma T \tag{3.36}$$

The coefficient of bulk viscosity is given by

$$\xi = \left[\frac{r(1-\in_0)-1}{W}\right] Co\sec h\, 2\gamma T \tag{3.37}$$

The generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\gamma}{3} Tanh 2\gamma T \tag{3.38}$$

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{W}{\gamma}\right) Cosh 2\gamma T\right]^{\frac{1}{2}} dT^2 - \left[\left(\frac{W}{\gamma}\right) Cosh 2\gamma T\right](d\theta^2 + Cosh^2\theta\, d\varphi^2) - \left[\left(\frac{\gamma}{W}\right) Sech 2\gamma T\right](d\psi + Sinh\theta\, d\varphi)^2 \tag{3.39}$$

Thus (3.39) together with (3.15), (3.32), (3.33), (3.35) & (3.37) constitutes a Bianchi type-VIII string cosmological model with bulk viscosity in Saez-Ballester (1986) theory of gravitation.

Bianchi type-IX ($\delta = 1$) cosmological model:

If $\delta = 1$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 + (r-1) S^{2n+2} - \frac{\omega}{2} k_1^{\,2} = 0 \tag{3.40}$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

For $n = -1$, with a suitable substitution from equation (3.39), we get

$$S'^2 = \gamma^2 S^2 - W^2 S^6 \tag{3.41}$$

where $\gamma^2 = \frac{2(1-r) + \omega k_1^{\,2}}{2}$ & $W^2 = \frac{1}{4}$.

From equation (3.41), we get

$$S^2 = \left(\frac{\gamma}{W}\right) Sech 2\gamma T \tag{3.42}$$

From equations (3.11) & (3.42), we get

$$R^{-2} = \left(\frac{\gamma}{W}\right) Sech 2\gamma T \tag{3.43}$$

From equations (3.8), (3.15), (3.42) & (3.43) we get the energy density

$$\rho = r \frac{\gamma}{W} Sech 2\gamma T \tag{3.44}$$

From equations (3.6), (3.15), (3.42) & (3.43) we get the total pressure

$$\overline{p} = (r-1) \frac{\gamma}{W} Sech 2\gamma T \tag{3.45}$$

The proper pressure is given by

$$p = \in_0 \rho = r \in_0 \frac{\gamma}{W} Sech 2\gamma T \qquad (3.46)$$

From equations (3.7), (3.15), (3.42) & (3.45) we get
the string tension density

$$\lambda = \frac{\gamma}{W} Sech 2\gamma T \qquad (3.47)$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = (r-1)\frac{\gamma}{W} Sech 2\gamma T \qquad (3.48)$$

The coefficient of bulk viscosity is given by

$$\xi = \left[\frac{r(1-\in_0)-1}{W}\right] Co\sec h\, 2\gamma T \qquad (3.49)$$

The generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\gamma}{3} Tanh 2\gamma T \qquad (3.50)$$

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{W}{\gamma}\right) Cosh 2\gamma T\right]^{\frac{1}{2}} dT^2 - \left[\left(\frac{W}{\gamma}\right) Cosh 2\gamma T\right](d\theta^2 + Sin^2\theta\, d\varphi^2) - \left[\left(\frac{\gamma}{W}\right) Sech 2\gamma T\right](d\psi + Cos\theta\, d\varphi)^2 \qquad (3.51)$$

Thus (3.51) together with (3.15), (3.44), (3.45), (3.47) & (3.49) constitutes a Bianchi type-IX string cosmological model with bulk viscosity in Saez-Ballester (1986) theory of gravitation.

ISOTROPIC COSMOLOGICAL MODELS:

Bianchi type-II ($\delta = 0$) cosmological model:

If $\delta = 0$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 - \frac{\omega}{2} k_1^{\ 2} = 0 \qquad (3.52)$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

For $n = 1$, with a suitable substitution from equation (3.52), we get

$$S'^2 = W^2 S^6 - \gamma^2 S^2 \qquad (3.53)$$

where $W^2 = \frac{1-4r}{12}, r < 0$ & $\gamma^2 = \frac{\omega k_1^{\ 2}}{6}$.

From equation (3.53), we get

$$S^2 = R^2 = \left(\frac{\gamma}{W}\right) Sec 2\gamma T \qquad (3.54)$$

From equations (3.8), (3.15) & (3.54) we get

the energy density

$$\rho = -r \frac{W}{\gamma} Cos 2\gamma T \qquad (3.55)$$

From equations (3.6), (3.15) & (3.54) we get

the total pressure

$$\bar{p} = \frac{(r-1)}{3} \frac{W}{\gamma} Cos 2\gamma T \qquad (3.56)$$

The proper pressure is given by

$$p = \in_0 \rho = -r \in_0 \frac{W}{\gamma} Cos 2\gamma T \qquad (3.57)$$

From equations (3.7), (3.15), (3.54) & (3.56) we get

the string tension density

$$\lambda = -\frac{W}{\gamma} Cos 2\gamma T \tag{3.58}$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = (1-r)\frac{W}{\gamma} Cos 2\gamma T \tag{3.59}$$

The coefficient of bulk viscosity is given by

$$\xi = \left[\frac{1-r(1+3\epsilon_0)}{9}\right]\frac{W}{\gamma^2}\, Cos\, 2\gamma T\, Cot\, 2\gamma T \tag{3.60}$$

The generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \gamma Tan 2\gamma T \tag{3.61}$$

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right]^{\frac{3}{2}} dT^2 - \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right][(d\theta^2 + d\varphi^2) + (d\psi + \theta\, d\varphi)^2] \tag{3.62}$$

Thus (3.62) together with (3.15), (3.55), (3.56), (3.58) & (3.60) constitutes a Bianchi type-II string cosmological model with bulk viscosity in isotropic form in Saez-Ballester (1986) theory of gravitation.

Bianchi type-VIII ($\delta = 0$) cosmological model:

If $\delta = -1$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 - (r-1) S^{2n+2} - \frac{\omega}{2} k_1{}^2 = 0 \tag{3.63}$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

For $n = 1$, with a suitable substitution, from equation (3.63), we get

$$S'^2 = W^2 S^6 - \gamma^2 S^2 \tag{3.64}$$

where $W^2 = \frac{5-8r}{12}$, $r < 0$ & $\gamma^2 = \frac{\omega k_1^{\,2}}{6}$.

From equation (3.64), we get

$$S^2 = R^2 = \left(\frac{\gamma}{W}\right) Sec 2\gamma T \tag{3.65}$$

From equations (3.8), (3.15) & (3.65) we get

the energy density

$$\rho = -2r \frac{W}{\gamma} Cos 2\gamma T \tag{3.66}$$

From equations (3.6), (3.15) & (3.65) we get

the total pressure

$$\bar{p} = 2\frac{(r-1)}{3}\frac{W}{\gamma} Cos 2\gamma T \tag{3.67}$$

The proper pressure is given by

$$p = \in_0 \rho = -2r \in_0 \frac{W}{\gamma} Cos 2\gamma T \tag{3.68}$$

From equations (3.7), (3.15), (3.65) & (3.67) we get

the string tension density

$$\lambda = -2\frac{W}{\gamma} Cos 2\gamma T \tag{3.69}$$

The particle energy density is given by

$$\rho_p = \rho - \lambda = 2(1-r)\frac{W}{\gamma} Cos 2\gamma T \tag{3.70}$$

The coefficient of bulk viscosity is given by

$$\xi = \left[\frac{-2[1+r(3\in_0 -1)]}{9}\right]\frac{W}{\gamma^2} Cos\, 2\gamma T\, Cot 2\gamma T \tag{3.71}$$

The generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \gamma Tan 2\gamma T \tag{3.72}$$

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right]^{\frac{3}{2}} dT^2 - \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right][(d\theta^2 + Cosh^2\theta\, d\varphi^2) + (d\psi + Sinh\theta\, d\varphi)^2] \tag{3.73}$$

Thus (3.73) together with (3.15), (3.66), (3.67), (3.69) & (3.71) constitutes a Bianchi type-VIII string cosmological model with bulk viscosity in isotropic form in Saez-Ballester (1986) theory of gravitation.

Bianchi type-IX ($\delta = 1$) cosmological model:

If $\delta = 1$, from equation (3.14) we get

$$C_1 \frac{S''}{S} - C_2 \frac{S'^2}{S^2} - \frac{(4r-1)}{4} S^4 + (r-1) S^{2n+2} - \frac{\omega}{2} k_1^{\ 2} = 0 \tag{3.74}$$

where $C_1 = r(n-1)$,

$C_2 = r(n-1) + n(n+2)$.

For $n = 1$, with a suitable substitution, from equation (3.74), we get

$$S'^2 = \gamma^2 S^2 - W^2 S^6 \tag{3.75}$$

where $W^2 = \frac{1}{4}$ & $\gamma^2 = \frac{-\omega k_1^{\ 2}}{6}, \omega < 0$.

From equation (3.75), we get

$$S^2 = R^2 = \left(\frac{\gamma}{W}\right) Sech\, 2\gamma T \qquad (3.76)$$

From equations (3.6) to (3.8), (3.15) & (3.76), we can observe that the energy density ρ, the total pressure $\bar{p}$, the string tension density λ and the coefficient of bulk viscosity ξ will vanish.

The metric (2.1), in this case can be written as

$$ds^2 = \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right]^{\frac{3}{2}} dT^2 - \left[\left(\frac{\gamma}{W}\right) Sec 2\gamma T\right] [(d\theta^2 + Sin^2\theta\, d\varphi^2) + (d\psi + Cos\theta\, d\varphi)^2] \qquad (3.77)$$

Thus (3.77) together with (3.15) & (3.76) constitutes a Bianchi type-IX vacuum cosmological model in isotropic form in Saez-Ballester (1986) theory of gravitation.

4. Some other important properties of the Model:

Bianchi type-II cosmological model ($\delta = 0$):

- The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = \left[\left(\frac{\gamma}{W}\right) Sec\, 2\gamma T\right]^{\frac{2n+1}{2}} \qquad (4.1)$$

- The average scale factor for the model is

$$a = V^{\frac{1}{3}} = \left[\left(\frac{\gamma}{W}\right) Sec\, 2\gamma T\right]^{\frac{2n+1}{6}} \qquad (4.2)$$

- The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{;i} = -(2n+1)\,\gamma Tan\, 2\gamma T \qquad (4.3)$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{(2n+1)^2}{2}\gamma^2 Tan^2\, 2\gamma T \qquad (4.4)$$

- **Deceleration Parameter:**

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^i + \frac{1}{3}\theta^2) = -\left(\frac{6}{2n+1}\right)Cosec^2\, 2\gamma T - 1 \qquad (4.5)$$

From equation (4.5), we can observe that the deceleration parameter q is always negative for $n > 0$ and hence it represents accelerating universe.

- The average anisotropy parameter is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2(n-1)^2}{(2n+1)^2} \qquad (4.6)$$

where $\Delta H_i = H_i - H \quad (i = 1,2,3)$.

- **Jerk Parameter:**

Jerk parameter in cosmology is defined as the dimensionless third derivative of scale factor with respect to cosmic time and is given by

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a} \qquad (4.7)$$

We can rewrite equation (4.6) as

$$j = q + 2q^2 - \frac{\dot{q}}{H} \qquad (4.8)$$

where q is deceleration parameter.

Hence the expression for jerk parameter is given by

$$j(t)=\frac{9(4n+10)}{(2n+1)^2}Co\sec^2 2\gamma T+1 \qquad (4.9)$$

- **Age of the universe:** The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at redshift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (4.2) ,we get

$$1+z=\frac{a_0}{a}=\left(\frac{\gamma/w\, Sec2\gamma T_0}{\gamma/w\, Sec2\gamma T}\right)^{2n+1/6} \qquad (4.10)$$

The above equation gives

$$Cos2\gamma T=(1+z)^{6/2n+1}Cos2\gamma T_0 \qquad (4.11)$$

Taking limit $z\to\infty$ in (4.11), we get the age of the universe is

$$T_0=\frac{\pi}{4k_1}\sqrt{\frac{2n(n+2)}{\omega}} \qquad (4.12)$$

which is different from the present estimate i.e. $T_0 = 14Gyr$. But if we take different values of $n\,\&\,\omega$ with $k_1 = 1$, then the derived model is in good agreement with the present age of the universe as shown in Fig.1.

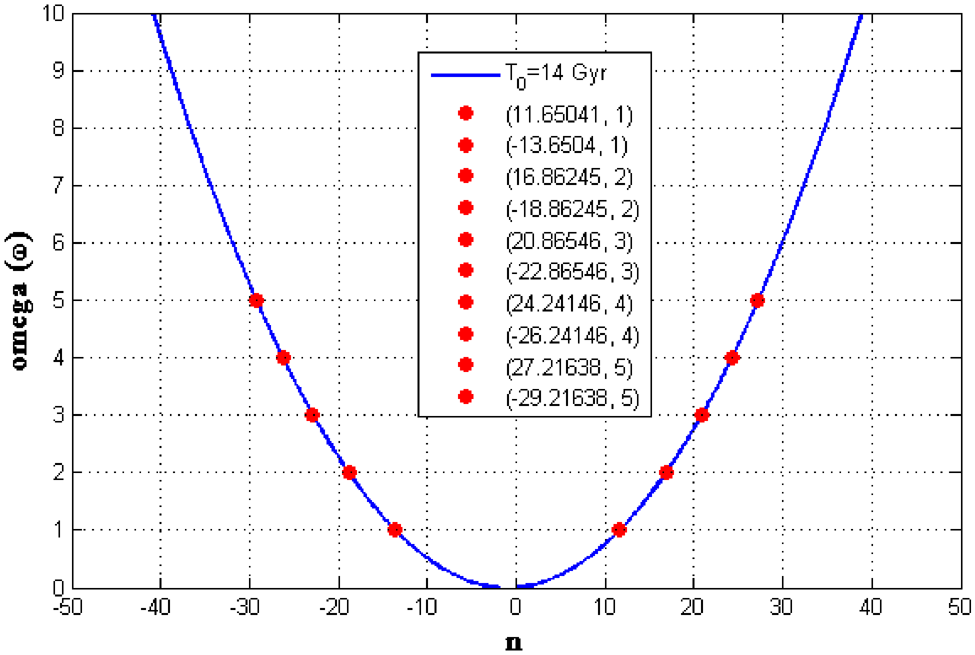

Fig. 1: Plot of age of the universe.

Bianchi type-VIII ($\delta = -1$) & IX ($\delta = 1$) cosmological models:

- The spatial volume for both the models (3.39) & (3.51) is

$$V = \left[\left(\frac{\gamma}{W}\right) Sech 2\gamma T\right]^{\frac{-1}{2}} f(\theta) \tag{4.13}$$

where $\gamma^2 = \dfrac{2(r-1)+\omega k_1^{\,2}}{2}$ & $W^2 = \dfrac{1}{4}$, $\quad \gamma^2 = \dfrac{2(1-r)+\omega k_1^{\,2}}{2}$ & $W^2 = \dfrac{1}{4}$ and

$f(\theta) = Cosh\theta$ & $Sin\theta$ for Bianchi type-VIII & IX respectively.

- The average scale factor for both the models (3.39) & (3.51) is

$$a = V^{1/3} = \left[\left(\frac{\gamma}{W}\right) Sech 2\gamma T\right]^{\frac{-1}{6}} [f(\theta)]^{1/3} \tag{4.14}$$

- The expression for expansion scalar θ and the shear scalar σ for the models (3.39) & (3.51) are given by

$$\theta = \gamma\, Tanh\, 2\gamma T \quad (4.15)$$

$$\sigma^2 = \frac{\gamma^2}{2} Tanh^2\, 2\gamma T \quad (4.16)$$

- **Deceleration Parameter:**

  The deceleration parameter q for the models (3.39) & (3.51) is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -6\,Co\sec h^2\, 2\gamma T - 1 \quad (4.17)$$

From equation (4.17), we can observe that the deceleration parameter q is negative for $T \neq 0$ and hence they represent accelerating universe.

- The average anisotropy parameter for the models (3.39) & (3.51) are defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 8 \text{ , where } \Delta H_i = H_i - H \ \ (i = 1,2,3) \quad \backslash \quad (4.18)$$

- **Jerk Parameter:**

  The expression for jerk parameter is given by

$$j(t) = -54\,Co\sec h^2\, 2\gamma T - 1 \quad (4.19)$$

- **Age of the universe:** The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at redshift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1 + z = \frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (4.14) ,we get

$$1 + z = \frac{a_0}{a} = \left(\frac{\gamma/_w\, Sech 2\gamma T_0}{\gamma/_w\, Sech 2\gamma T}\right)^{-1/6} \quad (4.20)$$

The above equation gives

$$Cosh2\gamma T = Cosh2\gamma T_0 (1+z)^{-6} \tag{4.21}$$

Taking limit $z \to \infty$ in (4.21), we get $T_0 = \frac{\pi}{4}\sqrt{\frac{2}{2(r-1)+\omega k_1^{\,2}}}$ (4.22)

By giving suitable values to $r, k_1 \,\&\, \omega$ in (4.22), we can always get the present age of the universe.

Bianchi type- II & VIII Isotropic cosmological models:

- The spatial volume for the models is

$$V = \left[\left(\frac{\gamma}{W}\right) Sec2\gamma T\right]^{\frac{3}{2}} f(\theta) \tag{4.23}$$

where $\gamma^2 = \frac{\omega k_1^{\,2}}{6}, W^2 = \frac{1-4r}{12} \,\&\, f(\theta) = 1$ for Bianchi type- II metric,

$\gamma^2 = \frac{\omega k_1^{\,2}}{6}, W^2 = \frac{5-8r}{12} \,\&\, f(\theta) = Cosh\theta$ for Bianchi type- VIII metric.

- The average scale factor for the models is

$$a = V^{1/3} = \left[\left(\frac{\gamma}{W}\right) Sec2\gamma T\right]^{\frac{1}{2}} [f(\theta)]^{1/3} \tag{4.24}$$

- The expression for expansion scalar θ and the shear scalar σ for the models are given by

$$\theta = 3\gamma\, Tan2\gamma T \tag{4.25}$$

$$\sigma^2 = \frac{9\gamma^2}{2} Tan^2 2\gamma T \tag{4.26}$$

- **Deceleration Parameter:**

The deceleration parameter q for the models is given by

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=-2Co\sec^2 2\gamma T-1 \tag{4.27}$$

From equation (4.27), we can observe that the deceleration parameter q is negative for $T\neq 0$ and hence they represent accelerating universe.

- The average anisotropy parameter for the models are defined by

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2=0 \text{ , where } \Delta H_i=H_i-H \ \ (i=1,2,3) \tag{4.28}$$

- **Jerk Parameter:**

The expression for jerk parameter is given by

$$j(t)=14Co\sec^2 2\gamma T+1 \tag{4.29}$$

- **Age of the universe:** The look-back time, $\Delta t=t_0-t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (4.24) ,we get

$$1+z=\frac{a_0}{a}=\left(\frac{\gamma/_w\, Sec2\gamma T_0}{\gamma/_w\, Sec2\gamma T}\right)^{1/2} \tag{4.30}$$

The above equation gives

$$Cos2\gamma T=Cos2\gamma T_0(1+z)^2 \tag{4.31}$$

Taking limit $z \to \infty$ in (4.31), we get $T_0 = \frac{\pi}{4}\sqrt{\frac{6}{\omega k_1^2}}$ (4.32)

By giving suitable values to k_1 & ω in (4.32), we can always get the present age of the universe.

- The vorticity tensor $w_{ij} = u_{i,j} - u_{j,i}$ is a measure of the rotation of the local rest-frame relative to the compass of inertia, is identically zero. Hence the fluid filling the universes are non-rotational.

5. Conclusions:

In this paper we have presented homogeneous Bianchi type-II, VIII & IX anisotropic as well as isotropic bulk viscous string cosmological models in a theory of gravitation proposed by Saez and Ballester (1986).

In standard cosmology, data tells us that the universe is homogeneous and isotropic. However, after the discovery of temperature anisotropies of the Cosmic Microwave background (CMB) radiation, it is conjectured that there are unobservable small amount of anisotropies are present in the early stages of evolution of the universe. The presence of this feature seems to be inconsistent with isotropic FRW model. The current CMB data supports an inflationary big bang model of cosmic origin for our universe. Hence in our model there is an unobservable small amount of anisotropy present. The corresponding studies to calculate bounds on the cosmological parameters do exist. Of course this will lead to discussion of dark energy via viscosity. For this one can refer the article by Bamba et al. (2012).

The following are the observations and conclusions:

	Anisotropic universe		Isotropic universe
	Bianchi type-II metric	Bianchi type-VIII &IX metrics	Bianchi type –II & VIII metrics
Singularity	Initial singularity at T=0 for $n>0$	No singularities	Singularities at $T=\frac{(2r+1)\pi}{4\gamma}$, $r=0, \pm 1, \pm 2, \ldots$
Spatial volume V	Varies with time T and θ	Varies with time T and θ	Varies with time T and θ
Expansion scalar θ	Increases with the increase of time	Increases with the increase of time	Increases with the increase of time
Shear scalar σ	Increases with the increase of time	Increases with the increase of time	Increases with the increase of time
Hubble parameter H	Increases with the increase of time	Increases with the increase of time	Increases with the increase of time
Energy density ρ, Total pressure $\bar{p}$, String density λ & Coefficient of bulk viscosity ξ	Tends to unity as $T\to 0$ and zero as $T\to\infty$	Tends to unity as $T\to 0$ and zero as $T\to\infty$	Tends to unity as $T\to 0$ and zero as $T\to\infty$
Deceleration parameter q	$q<0$ Accelerating	$q<0$ Accelerating	$q<0$ Accelerating

It is well known that scalar field and bulk viscosity have a significant role in getting an accelerated universe. The anisotropic as well as isotropic exact models presented here are new, more general and represent not only the early stage of evolution but also the present universe.

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