Strange quark matter attached to string cloud in general scalar tensor theory of gravitation

V U M Rao and D Neelima

Department of Applied Mathematics, Andhra University, Visakhapatnam, A.P., India

Abstract

Bianchi type-VI_0 space time with strange quark matter attached to string cloud in Nordtvedt [1] general scalar tensor theory of gravitation with the help of a special case proposed by Schwinger [2] is obtained. The field equations have been solved by using the anisotropy feature of the universe in the Bianchi type-VI_0 space time. Some important features of the model, thus obtained, have been discussed.

Keywords: Bianchi type-VI_0, quark matter, cosmic strings, general scalar tensor theory of gravitation.

1. Introduction

Nordtvedt [1] proposed a general class of scalar-tensor gravitational theories in which the parameter ω of the BD theory is allowed to be an arbitary (positive definite) function of the scalar field ($\omega \rightarrow \omega(\varphi)$). The study of scalar field cosmological models in the frame work of Nordtvedt's theory has attracted many research workers. This general class of scalar-tensor gravitational theories includes the Jordan [3] and Brans-Dicke [4] theories as special cases. This general cases of scalar-tensor theories would seen to lead to a super richness, or arbitrariness, of possible theories.

The field equations of general scalar-tensor theory proposed by Nordtvedt are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\varphi^{-1}T_{ij} - \omega\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) - \varphi^{-1}\left(\varphi_{;j} - g_{ij}\varphi_{;k}^{,k}\right), \quad (1)$$

$$\varphi_{;k}^{,k} = \frac{8\pi T}{3+2\omega} - \frac{1}{(3+2\omega)}\frac{d\omega}{d\varphi}\varphi_{,i}\varphi^{,i} , \quad (2)$$

where R_{ij} is the Ricci tensor, R is the Curvature invariant, T_{ij} is the stress energy of the matter, comma and semicolon denote partial and covariant differentiation respectively. (We use gravitational units $8\pi G = C = 1$). Also, we have

$$T_{;j}^{ij} = 0 , \quad (3)$$

which is a consequence of the field equations (1) and (2).

Dutta Choudary and Battacharya [5] have shown that Birkhoff's theorem holds, both in vacuum as well as in the presence of an electromagnetic field, for a general class of scalar-tensor theories proposed by Nordtvedt when the scalar field is time independent. In contrast, Banerjee and Dutta choudary [6] have discussed the static gravitational and Maxwell fields in this theory and have also obtained static plane symmetric exact solutions in the theory with ω given by Barker's form. Several investigations have been made in higher dimensional cosmology in the framework of different scalar-tensor theories of gravitation. In particular, Reddy and Naidu [7], Reddy et al. [8], Reddy and Naidu [9], Reddy and Naidu [10] have studied the higher dimensional string cosmological models in Brans-Dicke and other scalar-tensor theories of gravitation. Barker [11], Ruban and Finkelstein [12], Banerjee and Santos [13, 14], Shanti [15] and Shanti and Rao [16] are some of the authors who have investigated several aspects of Nordtvedt general scalar-tensor theory. Rao and Sreedevi Kumari [17] have discussed a cosmological model with negative constant deceleration parameter in a general scalar tensor theory of gravitation and Rao et al. [18] have obtained Kaluza-Klein radiating model in a general scalar tensor theory of gravitation. Recently, Rao and Neelima [19- 21] have obtained dark energy, LRS Bianchi type-I dark energy and Kantowski-Sachs string with bulk

viscosity in general scalar tensor theory of gravitation.

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Because, one of such transitions during the phase transitions of the universe could be Quark Glucon Plasma (QGP) harden gas (called quark-hadron phase transition) when cosmic temperature is $T \approx 200$ MeV. Strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). In the framework of this model, the quark matter is composed of massless u and d quarks, massive S quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are massless and non-interacting. Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3} \quad , \tag{4}$$

Where ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c \quad , \tag{5}$$

But the total pressure is

$$p = p_q - B_c \quad . \tag{6}$$

For more information and review of strange quark matter attached to string cloud, one can refer to Adhav et al. [22]. Khadekar et al. [23] have confined their work to the quark matter attached to the topological defects in general relativity. Khadekar and Rupali Wanjari [24] have discussed geometry of quark and strange quark matter in higher dimensional general relativity. Rao and Neelima [25] have discussed axially symmetric space time with strange quark matter attached to the string cloud in self creation theory and general relativity and established that the additional condition, special law of variation of Hubble parameter proposed by Bermann [26], taken by Katore and Shaikh [27] in general relativity is superfluous. Recently, Rao and Sireesha [28, 29] have discussed axially symmetric and Bianchi type-II, VIII & IX space times with strange quark matter attached to the string cloud respectively in Brans-Dicke theory of gravitation.

In this paper, we study the Bianchi type-VI$_0$ strange quark matter attached to the string cloud in general scalar tensor theory of gravitation.

2. Metric and energy momentum tensor
We consider the Bianchi type–VI$_0$ line element in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2 \quad , \tag{7}$$

where $A, B,$ and C are the functions of time t only.
The energy momentum tensor for string cloud [30] is given by

$$T_{ij} = \rho\, u_i\, u_j - \rho_S x_i x_j \quad . \tag{8}$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_S is the string tension density. They are related by

$$\rho = \rho_p + \rho_S \quad , \tag{9}$$

where ρ_p is the particle energy density. We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, here we will take quarks instead of particles in the string cloud. Hence, we consider quark matter energy density instead of particle energy density in the string cloud.
In this case from (9), we get

$$\rho = \rho_q + \rho_S + B_c \quad . \tag{10}$$

From eqs. (9) and (10), we have energy momentum tensor for strange quark matter attached to the string cloud [31] as

$$T_{ij} = (\rho_q + \rho_S + B_c)u_i\, u_j - \rho_S x_i\, x_j \quad , \tag{11}$$

where u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string. u_i and x_i are

$$u_i u^i = -x_i x^i = 1 \quad \text{and} \quad u^i x_i = 0 \tag{12}$$

We have taken the direction of string along x-axes. Then the components of energy momentum tensor are

$$T_1^1 = \rho_S , T_2^2 = T_3^3 = 0, T_4^4 = \rho \quad . \tag{13}$$

Where ρ and ρ_s are functions of t only.

3. Solving the field equations
Using commoving coordinates, the field equations (1), (2) for the metric (7) with the help of equations (10) to (13) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} + \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{\varphi}\rho_S \quad , \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} + \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = 0 \quad , \tag{15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \quad , \tag{16}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} - \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{\varphi}\rho \quad , \tag{17}$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \quad , \tag{18}$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi(\rho + \rho_S)}{(3+2\omega)} - \frac{\dot{\varphi}^2}{(3+2\omega)}\frac{d\omega}{d\varphi} \quad , \tag{19}$$

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \rho_S\left(\frac{\dot{A}}{A}\right) = 0 \quad . \tag{20}$$

Here the overhead dot denotes differentiation with respect to t. From eq. (18), we get $C = \alpha B$.

Without loss of generality, we can take $\alpha = 1$, so that we have

$$C = B \quad . \tag{21}$$

Using (21), the field equations (14) to (20) reduce to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} + \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(2\frac{\dot{B}}{B}\right) = \frac{8\pi}{\varphi}\rho_S \quad , \tag{22}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \quad , \tag{23}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} - \frac{1}{2}\frac{\omega\dot{\varphi}^2}{\varphi^2} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{8\pi}{\varphi}\rho \quad . \tag{24}$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{8\pi(\rho + \rho_S)}{(3 + 2\omega)} - \frac{\dot{\varphi}^2}{(3 + 2\omega)}\frac{d\omega}{d\varphi} \quad , \tag{25}$$

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) - \rho_S\left(\frac{\dot{A}}{A}\right) = 0 \quad . \tag{26}$$

By using the transformation $dt = AB^2 dT$, the above field equations (22) to (26) can be written as

$$2\frac{B''}{B} - 3\frac{B'^2}{B^2} - 2\frac{A'B'}{AB} + B^4 + \frac{\omega}{2}\frac{\varphi'^2}{\varphi^2} + \frac{\varphi''}{\varphi} - \frac{\varphi'}{\varphi}\frac{A'}{A}$$
$$= \frac{8\pi}{\varphi}\rho_S\left(A^2 B^4\right), \tag{27}$$

$$\frac{A''}{A} - \frac{A'^2}{A^2} + \frac{B''}{B} - 2\frac{B'^2}{B^2} - 2\frac{A'B'}{AB} - B^4 + \frac{\omega}{2}\frac{\varphi'^2}{\varphi^2}$$
$$+ \frac{\varphi''}{\varphi} - \frac{\varphi'}{\varphi}\frac{B'}{B} = 0, \tag{28}$$

$$2\frac{A'B'}{AB} + \frac{B'^2}{B^2} - B^4 - \frac{\omega}{2}\frac{\varphi'^2}{\varphi^2} + \frac{\varphi'}{\varphi}\left(\frac{A'}{A} + 2\frac{B'}{B}\right)$$
$$= \frac{8\pi}{\varphi}\rho\left(A^2 B^4\right) \quad , \tag{29}$$

$$(3 + 2\omega)\varphi'' = 8\pi(\rho + \rho_S)\ (A^2 B^4) - \varphi'^2 \frac{d\omega}{d\varphi} \quad , \tag{30}$$

$$\rho' + \rho\left(\frac{A'}{A} + 2\frac{B'}{B}\right) - \rho_S\left(\frac{A'}{A}\right) = 0 \quad . \tag{31}$$

Hereafter, the over head dash denotes differentiation with respect to T.

The field equations (27) to (30) are four independent equations with six unknowns $A, B, \omega, \varphi, \rho$ and ρ_S.

From (27), (29) and (30), we get

$$2(1 + \omega)\varphi'' - 2\varphi'\left(\frac{B'}{B}\right) - 2\varphi\left(\frac{B'}{B}\right)' + \varphi'^2\frac{d\omega}{d\varphi} = 0 \quad . \tag{32}$$

Since ω is a function of ϕ, we obtain strange quark matter attached to string cloud in this theory with a special case proposed by Schwinger, i.e.

$$3 + 2\omega(\varphi) = \frac{1}{\lambda\varphi}, \quad \lambda = \text{constant} \quad , \tag{33}$$

From (32) and (33), we get

$$\varphi = (k_1 T + k_2)^2 \quad , \tag{34}$$

$$B = \frac{k_3}{(k_1 T + k_2)} \quad ,$$

where k_1, k_2 and k_3 are arbitrary constants. Without loss of generality we can take $k_3 = 1$, so that we have

$$B = \frac{1}{(k_1 T + k_2)} \quad . \tag{35}$$

From (28), (34) and (35), we get

$$A = (k_1 T + k_2)^{-1} \exp\left[\frac{k_1 T}{(k_1 T + k_2)} + \frac{k_4}{2k_1^2(k_1 T + k_2)^2}\right] \tag{36}$$

where $k_4 = 1 - \frac{k_1^2}{\lambda}$. The metric (7) can now be written as

$$ds^2 = (k_1 T + k_2)^{-6} \exp\left[\frac{2k_1 T}{(k_1 T + k_2)} + \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right] dT^2$$
$$(k_1 T + k_2)^{-2} \exp\left[\frac{2k_1 T}{(k_1 T + k_2)} + \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right]$$
$$e^{2x} dx^2 - (k_1 T + k_2)^{-2} e^{-2x}\left(dy^2 + dz^2\right) \tag{37}$$

From eq. (29), we get string energy density

$$\rho = \frac{k_5}{8\pi}(k_1 T + k_2)^4 \exp\left[\frac{-2k_1 T}{(k_1 T + k_2)} - \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right] , \tag{38}$$

where $k_5 = -\left(1 + \frac{k_1^2}{\lambda}\right) \succ 0, \quad i.e. \quad k_1^2 \prec -\lambda$.

In order to satisfy the energy conditions $\rho > 0$, $p > 0$, let us take $\lambda < 0$. From eq. (27), we get string tension density

$$\rho_S = \frac{-k_5}{8\pi}(k_1 T + k_2)^4 \exp\left[\frac{-2k_1 T}{(k_1 T + k_2)} - \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right]. \tag{39}$$

String particle density is

$$\rho_p = \rho - \rho_S =$$
$$\frac{k_5}{4\pi}(k_1 T + k_2)^4 \exp\left[\frac{-2k_1 T}{(k_1 T + k_2)} - \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right]. \tag{40}$$

Quark energy density is

$$\rho_q = \rho - B_c$$
$$= \frac{k_5}{8\pi}(k_1 T + k_2)^4 \exp\left[\frac{-2k_1 T}{(k_1 T + k_2)} - \frac{k_4}{k_1^2(k_1 T + k_2)^2}\right] - B_c , \tag{41}$$

Quark pressure is

$$p_q = \frac{\rho_q}{3}$$

$$= \frac{k_5}{24\pi}(k_1T+k_2)^4 \exp\left[\frac{-2k_1T}{(k_1T+k_2)} - \frac{k_4}{k_1{}^2(k_1T+k_2)^2}\right] - \frac{B_c}{3}$$

(42)

Hence, the metric (37) together with eqs. (34) and (38) to (42) represents Bianchi type-VI$_0$ strange quark matter attached to string cloud in general scalar tensor theory of gravitation.

4. Some important features of the model

The Volume element of the model (37) is given by

$$V = (-g)^{1/2} = (k_1T+k_2)^{-3}\exp\left[\frac{k_1T}{(k_1T+k_2)} + \frac{k_4}{2k_1{}^2(k_1T+k_2)^2}\right]$$

(43)

The expression for the expansion scalar θ is given by

$$\theta = u^i{}_{;i} = \frac{-1}{(k_1T+k_2)^2}\left[k_1(2k_2+3k_1T) + \frac{k_4}{k_1(k_1T+k_2)}\right]$$

(44)

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18(k_1T+k_2)^4}\left[k_1(2k_2+3k_1T) + \frac{k_4}{k_1(k_1T+k_2)}\right]^2$$

(45)

The deceleration parameter is given by

$$q = -3\theta^{-2}(\theta_{,i}u^i + \frac{1}{3}\theta^2) ,$$

$$q = -\left(\frac{3\left[\frac{3k_4}{(k_1T+k_2)^2} + \frac{k_1{}^2(3k_1T+k_2)}{(k_1T+k_2)}\right]}{\left[\frac{k_4}{k_1(k_1T+k_2)^2} + \frac{k_1(3k_1T+2k_2)}{(k_1T+k_2)}\right]^2} + 1\right)$$

(46)

The Hubble parameter is given by

$$H = \frac{-1}{3(k_1T+k_2)^2}\left[k_1(2k_2+3k_1T) + \frac{k_4}{k_1(k_1T+k_2)}\right]$$

(47)

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

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5. Conclusions

In this paper, we have presented homogeneous and anisotropic Bianchi type-VI$_0$ strange quark matter attached to string cloud in general scalar tensor theory of gravitation proposed by Nordtvedt with a special case proposed by Schwinger. The model (37) has initial singularity at $T = \frac{-k_2}{k_1}$. At this point of singularity, the spatial volume increases infinitely and vanishes as $T \to \infty$. Also we can observe that string energy density and string tension density increase with time. At $T = \frac{-k_2}{k_1}$ the expansion scalar θ and the shear scalar σ tend to infinity where as when $T \to \infty$, they tend to zero. Here, it is observed that $\rho + \rho_S = 0$ and hence we will get massive string case. The Hubble parameter H vanishes with the increase of time. The deceleration parameter appears with negative sign for large values of T, which implies the accelerated expansion of the universe and is consistent with the present day observations of type Ia supernovae (SN Ia). Also, since $\lim t_{T \to \infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy. We know that spatially homogeneous cosmological models will play an important role in attempts to understand the structure and properties of the space time. Moreover, from the theoretical point of view anisotropic universe has a greater generality than isotropic models. For our model, we get $\frac{\sigma}{\theta} \approx 0.6236$ which is greater than present upper limit $(10)^{-5}$ of $\frac{\sigma}{\theta}$ obtained by Collins *et al.* [32]. This fact implies that our solution also represents the early stages of evolution of the universe. Thus the cosmological model presented here is anisotropic, accelerating and non-rotating.

Acknowledgement

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