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Research note

Light scattering by cubical particle in the WKB approximation

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Abstract

In this work, we determined the analytical expressions of the form factor of a cubical particle in the WKB approximation. We adapted some variables (size parameter, refractive index, the scattering angle) and found the form factor in the approximation of Rayleigh-Gans-Debye (RGD), Anomalous Diffraction (AD), and determined the efficiency factor of the extinction. Finally, to illustrate our formalism, we analyzed some numerical examples.

Keywords: scattering, form factor, cubical particle , WKB approximation

1. Introduction

Electromagnetic scattering is the effect caused by EM waves hitting an object. The waves will then be scattered and the scattered field contains useful information about that object. Electromagnetic scattering happens in many situations, for example, sun light scattered by atmosphere, radio waves scattered by buildings or planes, and so on [1]. The study of EM wave scattering is of great interest and importance since it helps advancement in many different fields ranging from medical technology to computer engineering, geophysics, photonics, and military technology [2].

Solar radiation, penetrating the atmosphere, can interact with gases in the atmosphere, clouds, aerosols and soil. They can be absorbed or scattered back to space. Analysis of light scattering process by atmospheric particles based on Mie theory does not allow the clear physical interpretation because aerosols have various shapes (spherical, stars, rings, hexagonal, cubic ...) according to their forming conditions (temperature, humidity, supersaturation, ...) [3]. Mie theory, published in 1908, obtained a general rigorous solution, on the basis of electromagnetic theory, for the optical scattering by a homogeneous sphere with arbitrary size in a homogeneous medium, whatever the composition of the sphere and medium [4]. The extension to other forms of less ideal particles is revealed to be a difficult problem.

The methods of light scattering process analysis by the nonspherical particles are generally based on solving Maxwell's equations, either numerically or analytically. Examples of the numerical approach include the invariant embedding T-matrix approach [5], the digitized Green's-function method (DFG) [6], and an improved version of the Extended Boundary Condition Method (EBCM) [7]. These methods are flexible and general techniques for calculating the scattering and absorption characteristics of arbitrarily shaped particles, but they require a rather large computer data storage.

Solutions to the exact integral expression of scattering amplitude function are difficult. The function depends on the local field inside the particle and its permittivity. The local field is generally unknown. Therefore, certain approximations like Rayleigh, Born, Wentzel-Kramers-Brillouin (WKB) etc. are usually required to overcome the difficulty and avoid laborious computation [8].

In the WKB approximation, the internal field is equal to the incident field modulated by a phase delay factor which corresponds to an additional phase shift of the wave that propagates inside the particle. Therefore, the WKB approximation is a refinement of the Rayleigh-Debye-Gans approximation [9]. Because of the complexity of the general scattering theory for nonspherical particles, the possibility of applying the WKB approximation to modeling the scattering of light

by cubical objects is worth investigating. This approach is applied on spheres, cylinders and spheroid [10-12].
This work is devoted to a theoretical study of scattering of light by cubical particle, in the WKB approximation. Within the framework of the scattering theory, we investigate the form factor for this approximation which represents the modification of the scattered irradiance due to the finite size of the particle and to its deviation from sphericity, this factor allows us to determine amplitude of light scattering. The structure of the article is as follows: In the second section we derive a general analytical expression for the form factor expression for a cubical particle. In section three, RGD and AD are deduced from our general formula by varying some particle parameters. In the last section we determine the extinction coefficient. By using this formalism, numerical calculations are performed to illustrate the comportment of the form factor.

2. Form factor in WKB approximation

We present a general approach to calculate the form factor of structured particles. Suppose that the particle falls flat electromagnetic wave. We are using the integral representation of the form factor in the WKB approximation in a scalar form [12]:

$$F(\theta,\varphi)=\iiint_V \exp\left(ik.\vec{r}.\left(\vec{i}-\vec{o}\right)\right)\exp\left(ikW\right)dv' \ , \tag{1}$$

where $\vec{o}$ and $\vec{i}$ are the unit vectors along the directions of scattering and propagation of light, respectively, $\vec{r}$ is the radius vector of a point inside the particle, k is the free-space wave number, θ is the scattering angle, i.e., the angle between $\vec{i}$ and $\vec{o}$, φ is the azimuthally angle, v' is the volume of the scattered, and W is the optical path which is introduced by the scattering object expanded in the form

$$W=\int_{Ze}^{Z}\left[m\left(Z'\right)-1\right]dZ'=m(Z-Z_e) \ , \tag{2}$$

where, Z_e is the z-coordinates of the initial position of penetration of the object.

In the following, we investigate if the above method can be applied to the absorbing cubical particles, in order to estimate the intensity distribution of light scattered by the scattered.

We consider a Cartesian coordinate system, orthonormal $R(X,Y,Z)$ the origin coincides with the center of cube, we assume that an electromagnetic wave is incident in the plane YOZ on cube a homogeneous with an edge length α and m refractive index, the cube is positioned in such a way that two facets are illuminated by the incident light. They are aligned symmetrically with respect to the direction of incidence, which is perpendicular to the edge closest to the light source (the acute angle between the incident beam and each cube face is $\pi/4$), and the Z -axis is parallel to the incident ray.

In rectangular coordinates, the form factor can be written as

$$F(\theta,\varphi)=\iiint_{\vartheta}\exp ik(-x\sin\theta\cos\varphi-y\sin\theta\sin\varphi)$$
$$\exp ikz\left(m-\cos\theta\right)\exp-ikz_e(m-1)d\vartheta \ , \tag{3}$$

where x, y and z are the components of the position of the scattering element inside the object.

$$F(\theta,\varphi)=A(\theta,\varphi)\int_L ampl\left(Z_e\left(y\right),\theta\right)e^{-iky\sin\theta\sin\varphi}dy \ , \tag{4}$$

with L as the length of the particle on the Y-axis,

$$ampl\left(z_e\left(y\right),\theta\right)=\exp-iKz_e\left(y\right)\left(2m-1-\cos\theta\right)$$
$$-\exp-iKz_e\left(y\right)(1-\cos\theta) \ , \tag{5}$$

$$A(\theta,\varphi)=\frac{1}{ik(m-\cos(\theta))}\frac{\sin(d)}{d} \ , \tag{6}$$

and

$$d=\frac{kl}{2}\sin\theta\cos\varphi \ . \tag{7}$$

Equation (4) is obtained after integrating over x from $\frac{-a}{2}$ to $\frac{a}{2}$, and over z from $z_e(y)$ to $-z_e(y)$, where $z_e(y)$, and $-z_e(y)$ are the z-coordinates of the intersection of incident ray and the body surface.

We decompose the cube into two regions in figures (1-a) and (1-b). Each region in turn is divided into longitudinal slices (thickness dy and width Δz_j figure (1-b)). The form factor can be written as:

$$F(\theta,\varphi)=A(\theta,\varphi)\sum_{j=1}^{2}\int_{ymin_j}^{ymax_j} ampl_j$$
$$(z_{ej}\left(y\right),\theta)e^{-iky\sin\theta\sin\varphi}dy \ , \tag{8}$$

$$F(\theta,\varphi)=\sum_{j=1}^{2}F_j(\theta,\varphi) \ , \tag{9}$$

where $y\,min_j$ and $y\,max_j$ are the limits of the integral according to each region. Now, we just search $z_{ej}(y)$ expression in terms of y for each region $j=1,2$.

For region (1) defined by

$$-\sqrt{2}a\le y\le 0 \ , \tag{10}$$

the z-coordinates of the intersection of a line is parallel to k and the body surface is

$$z_{e1}=-\frac{a}{\sqrt{2}}-y \ . \tag{11}$$

After some algebraic manipulations, we obtain,

$$F_1(\theta,\varphi)=-i\frac{a^3}{2u}\frac{\sin d}{d}$$
$$\left(e^{ig}\frac{e^{-i(g-t)}-1}{-i(g-t)}-e^{iq}\frac{e^{-i(q-t)}-1}{-i(q-t)}\right) \ , \tag{12}$$

where

$$d=\frac{ka}{2}\sin\theta\cos\varphi \ , \tag{13}$$

$$t=\frac{ka}{\sqrt{2}}\sin\theta\sin\varphi \ , \tag{14}$$

$$g=u+\frac{\rho}{2} \ , \tag{15}$$

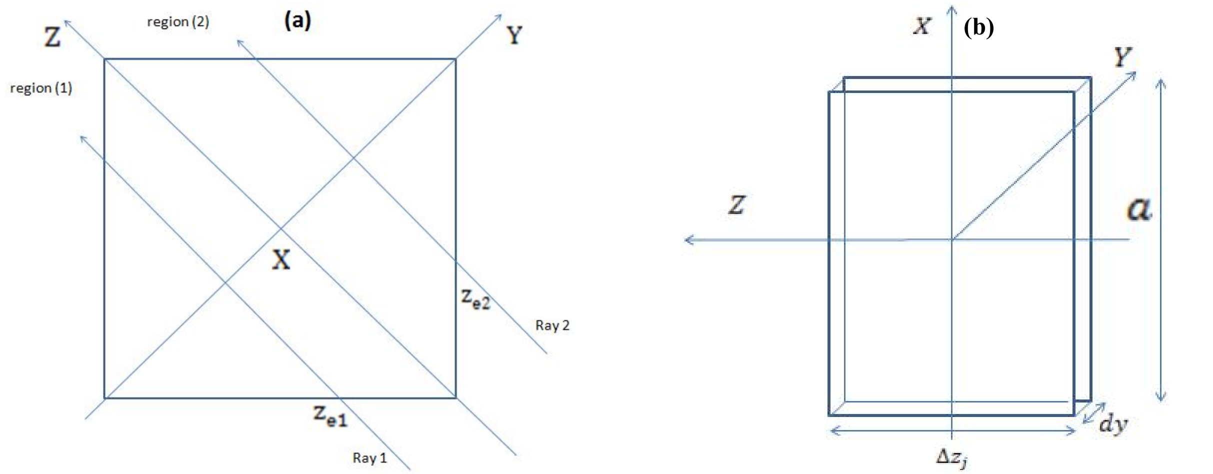

Figure 1. Decomposition of the cubical particle.

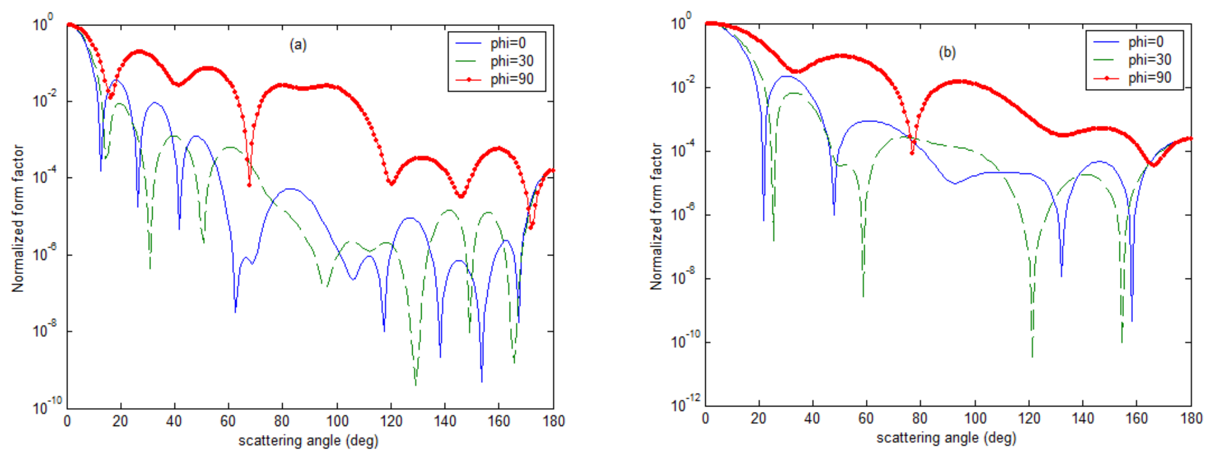

Figure 2. Normalized form factor as a function of scattering angle θ for absorbing cubical particle at a m=1.25+0.01.i for three values of φ, (a): $\frac{ka}{\sqrt{2}} = 10$; (b): $\frac{ka}{\sqrt{2}} = 6$.

$$q = -(u - \frac{\rho}{2}) , \tag{16}$$

$$\rho = \sqrt{2}ka(m-1) , \tag{17}$$

and

$$u = \frac{ka}{\sqrt{2}}(m - \cos\theta) . \tag{18}$$

For region (2) defined by

$$0 \le y \le \sqrt{2}a . \tag{19}$$

the z-coordinates is

$$z_{e2} = -\frac{a}{\sqrt{2}} + y , \tag{20}$$

we obtain

$$F_2(\theta,\varphi) = -i\frac{a^3}{2u}\frac{\sin d}{d} \left(e^{ig}\frac{e^{-i(g+t)} - 1}{-i(g+t)} - e^{iq}\frac{e^{-i(q+t)} - 1}{-i(q+t)} \right) . \tag{21}$$

We can see that, if we change φ by $\varphi \pm \pi$ i.e $(t \leftrightarrow -t)$ we have $F_1(\theta,\varphi) = F_2(\theta,\varphi \pm \pi)$ we can interpret this remark as contribution of region (2) or form factor at point $H(R,\theta,\varphi)$, the same as the contribution of region (1) at point $G(R,\theta,\varphi+\pi)$. This interpretation is done because of the symmetry of the hexagonal particles, where R is the distance from the observation point to the scattering object along the direction of the unit vector $\vec{o}$.

The form factor calculated is valid for any value of the phase delay of the wave penetrating through the center of the homogenous absorbing cubical particle. For illustration, we show in figure 2. the behavior of the form factor as a function of the scattering angle θ. This figure shows that the form factor exhibits some lobes with intensity decreasing with scattering angle, we find that the backscatter $(\theta > 90)$ is almost unnoticeable compared to the diffusion front which makes it around 10^4 times.

3. Special cases

As the WKB approximation is applicable for a wide

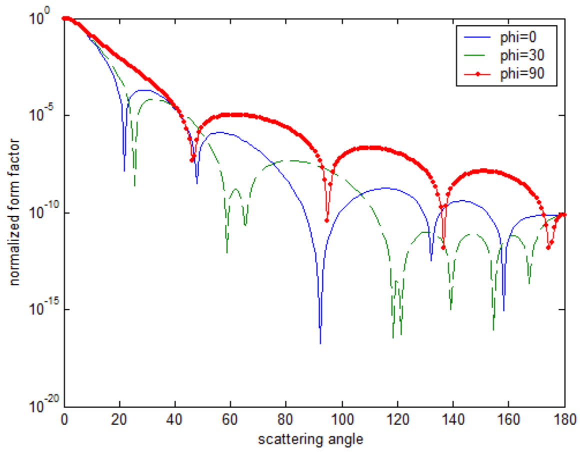

Figure 3. Normalized form factor in RGD approximation as a function of scattering angle θ for cubical particle.

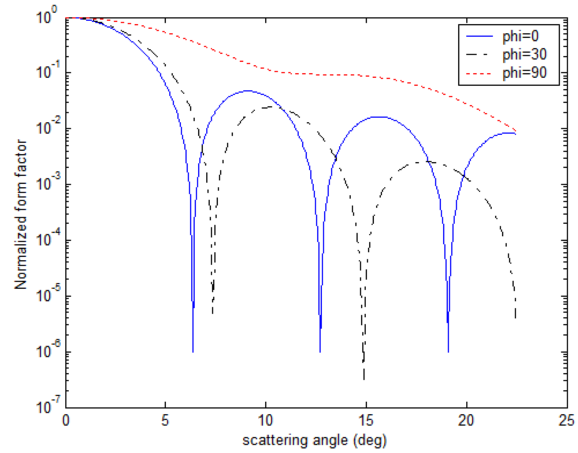

Figure 4. Normalized form factor as a function of scattering angle θ for cubical particle, for three values of $\boldsymbol{\varphi}$.

range of ρ, we can consider it the most general approach which includes the RGD and AD approximations. These approximations are applied for optically soft light scattering particles $|m-1| \ll 1$ [13-14]. So, from form factor in WKB which is the general formula, we can deduce the above approximations.

3. 1. RGD approximation

In the limit of a small refractive index, the WKB approximation reduces to RGD approximation. For this approximation, we assume that $\rho \ll 1$. The imaginary part of the form factor is null. We can write

$$\mathrm{F}(\theta,\varphi)=\frac{a^3}{u}\frac{sind}{d}\frac{cost-cosu}{u^2-t^2}\,. \qquad (22)$$

To illustrate our analytical results, we represent the behavior of the normalized form factor as a function of the scattering angle θ in the case of non absorbing cubical particle in figure 3. The parameters used in the calculation are $m=1.01$ and $\frac{\mathrm{ka}}{\sqrt{2}}=6$, for 3 values of $\boldsymbol{\varphi}$.

3. 2. Anomalous diffraction approximation

Such an approximation is valid if $\alpha = kd \gg 1$, where α is the size parameter, k is the free-space wave number, d is a geometrical path of a given ray through the considered nonspherical particle, and $|m-1| \ll 1$. This implies that the rays are not deviated when they cross the interface particle-medium and that the reflection at this interface is negligible. We consider in this case intermediary values of ρ and an angle $\theta \ll 1$.

We have,

$$u \approx \frac{\rho}{2}\,, \qquad (23)$$

$$t \approx \frac{ka}{\sqrt{2}}\theta sin\varphi\,, \qquad (24)$$

$$d \approx ka\theta cos\varphi\,. \qquad (25)$$

The expressions of the form factor are reduced to

$$F(\theta,\varphi)=-i\frac{a^3}{\rho}\frac{\mathrm{sind}}{d}\left(e^{-i\,t/2}\left(\frac{sin\left(\frac{\rho+t}{2}\right)}{\frac{\rho+t}{2}}\right.\right. \left.\left.+e^{i\,t/2}\frac{sin\left(\frac{\rho-t}{2}\right)}{\frac{\rho-t}{2}}\right)-2\frac{sint}{t}\right)\,. \qquad (26)$$

For illustration, we present in figure 4, the normalized form factor as a function of the scattering angle θ for three values of $\boldsymbol{\varphi}$. We have respected the condition imposed on the scattering angle, size parameter and refractive indece of the scattering object with $m = 1.1 + 0.01i$, and $\frac{\mathrm{ka}}{\sqrt{2}} = 25$.

4. Extinction efficiency

The extinction efficiency $\mathrm{Q_{ext}}$ is defined as the extinction cross section per unit of projected area of the particle on the plane perpendicular to the direction of the incoming beam. The extinction coefficient is expressed in terms of form factor P with air projected from the cube [12, 15, 16].

$$\mathrm{Q_{ext}}=\frac{\mathrm{k2\ Im}\left(\left|(\mathrm{m}-1)\mathrm{F}(0,0)\right|\right)}{\mathrm{P}}\,. \qquad (27)$$

The extinction efficiency of this case can be written as

$$\mathrm{Q_{ext}}=-2\mathrm{Re}\left(\frac{\mathrm{e}^{\mathrm{i}2\pi\frac{\sqrt{2}\mathrm{a}}{\lambda}(\mathrm{m}-1)}-1}{\mathrm{i}2\pi\frac{\sqrt{2}\mathrm{a}}{\lambda}(\mathrm{m}-1)}-1\right)\,. \qquad (28)$$

Figure 5 shows the extinction efficiencies of an cubicle particle as a function of $\mathrm{X}=\frac{\sqrt{2}\mathrm{a}}{\lambda}$, this parameter allows to compare the wavelength with the largest possible ray path within the considered particle, the imaginary part of refractive index is 0.01.

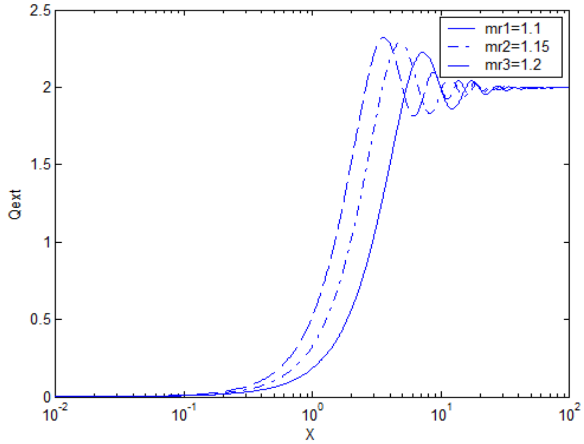

Figure 5. Evolution of the extinction efficiency as a function of $X = \frac{\sqrt{2}a}{\lambda}$ parameter for three different indices of refraction.

When the largest path is of the order of the wavelength, the extinction increases rapidly to a peak of maximum efficiency. Then, the efficiency decreases and oscillates around value 2.

When the real part increases, the amplitude of the maximum peak efficiency increases and shifts to the small particle. Furthermore, an increase in the number of oscillations is observed

Figure 6 shows the influence of particle absorption. The main influence of an increase in the absorption of the particle m_i, on the extinction coefficient is to damp the oscillations, in particular the amplitude of the maximum peak efficiency.

When studying in detail the evolution of the effectiveness of extinction, we note that extinction does not always increase when the absorption of the particle m_i increases. For example, in the case of small radii compared to the wavelength, the extinction increases as m_i increases. It is noted, however, that there is a fixed point, where extinction is independent of the value of the imaginary part of refractive index.

For real refractive index, the efficiency factor of the extinction becomes

$$Q_{ext} = 2\left(1 - \frac{\sin\left(2\pi\frac{\sqrt{2}a}{\lambda}(m-1)\right)}{2\pi\frac{\sqrt{2}a}{\lambda}(m-1)}\right). \quad (29)$$

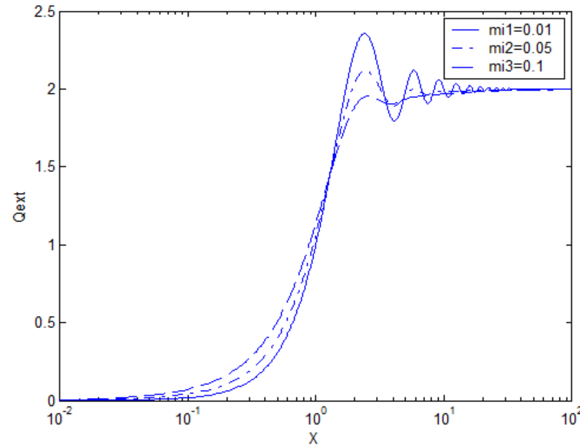

Figure 6. Evolution of the extinction efficiency as a function of $X = \frac{\sqrt{2}a}{\lambda}$ parameter, for three different indices of refraction.

This formula is consistent with the results found in Chylek and Klett [17].

5. Conclusion

In this article, we determined the form factor of a cubical particle in the WKB approximation, we limited the study to a particular case to illustrate the importance of this approximation. The technique of geometric decomposition of the particle to slices may also be useful to determine the form factor of the particles having an axis of symmetry for example hexagonal or parallelepiped. As a result of application of this technique, the formula for the form factor of light scattering by a cubical particle in the WKB approximation was obtained. We have obtained other approximations covered by the WKB approach. Indeed, by varying some particle parameters, the Rayleigh–Gans-Debye theory and the anomalous diffraction were deduced as particular cases. To illustrate the compartment of the form factor, some practical illustrations were analyzed in this work by using our analytical formulations.

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