Nodal quasi-particles of the high-T$_c$ superconductors as carriers of heat

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Abstract

In the quest for understanding correlated electrons, high-temperature superconductivity remains a formidable challenge and a source of insight. This paper briefly recalls the central achievement by the study of heat transport at low temperatures. At very low temperatures, nodal quasi-particles of the *d*-wave superconducting gap become the main carriers of heat. Their thermal conductivity is unaffected by disorder and reflects the fine structure of the superconducting gap. This finding had led to new openings in the exploration of other unconventional superconductors

1. Introduction

In spite of many years of intense research by a sizeable fraction of the condensed-matter physics community, high-T_c superconductivity remains a mystery[1]. A central question is the extent of the validity of Landau's Fermi liquid picture to describe the elementary excitations of the normal state. High-T_c cuprates are doped Mott insulators[2]. In other words, the parent compound is host to a particularly strong Coulomb repulsion, which should still be present when mobile carriers are added. It remains to be established, however, to what extent the strength of Coulomb repulsion becomes an obstacle for the formation of Landau quasi-particles in the zero-temperature limit.

Ironically, while the normal state of the high-T_c superconductivity remains a subject of intense controversy, the superconducting state appears less mysterious now than a decade before. Its exploration has led to a consensus on the *d*-wave symmetry of the order parameter[3]. The BCS theory of superconductivity applied to a *d*-wave superconductor appears to provide a successful explanation of a number of experimental features. In some cases this exploration has provided new insights for the investigation of other unconventional superconductors. This article focuses on the particular issue of heat transport at very low temperatures.

2. Nodal quasi-particles of a *d*-wave superconductor

In a nodal superconductor, the two-particle wave-function representing the superconducting order parameter changes sign along particular orientations in the reciprocal space. Consequently, the superconducting gap vanishes along these orientations. In a *d*-wave superconductor, the density of states in the vicinity of such a node is a linear function of energy. In such a configuration, the elementary excitations are massless and the quasi-particles are Dirac Fermions (See figure 1).

In a pure *d*-wave superconductor, there are no zero-energy excitations at nodes. According to the theory, however, even an infinitesimal amount of disorder breaks Cooper pairs and creates a finite density of states at zero energy. Therefore, both specific heat C, and thermal conductivity, κ, of a *d*-wave superconductor should present a finite temperature-linear term in the zero-temperature limit.

3. Thermal conductivity and specific heat

According to classical kinetics, in a gas-like system of particles, the amplitude of these two quantities are intimately linked through the equation:

$$\kappa = \frac{1}{3} C \upsilon \ell . \tag{1}$$

Here υ is the velocity and ℓ is the mean-free-path. Often, thermal conductivity cannot be interpreted in a straightforward manner. The variation of ℓ with temperature is not trivial and adds up to the particularities of the change in the density of states reflected in C. However at low enough temperature, the mean-free-path attains its maximum value and as the
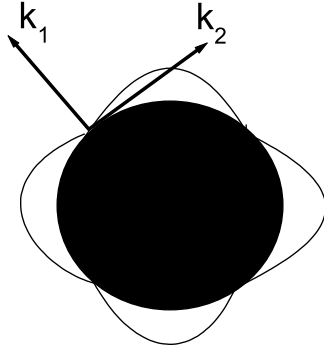
Figure 1. A d-wave superconducting gap around a two-dimensional circular Fermi surface. In the vicinity of each node, k_1 and k_2 are vectors normal and parallel to the Fermi surface.

velocity is also constant, both thermal conductivity and specific heat simply reflect the temperature dependence of the density-of-states, which is T-linear for Fermions and T^3 for Bosons.

There is, however, an important difference between these two measurable quantities. While only itinerant excitations participate in the transport of heat, the specific heat includes a contribution by local excitations. Therefore, only in the case of thermal conductivity the measured quantity directly reflects the presence of itinerant quasi-particles. superconducting gap in a d-wave superconductor.

4. Universal thermal conductivity

Heat is transported by both electrons and phonons. The separation between these two components of thermal conductivity is seldom straightforward. At low enough temperature, the phonon heat transport becomes ballistic. In other words, the mean-free-path of phonons attains the maximum value set by the finite dimensions of the sample. In this regime, κ_{ph} is expected to display a T^3 variation. Plotting the thermal conductivity divided by temperature as a function of T^2, one expects to have:

$$\frac{\kappa}{T} = a + bT^3 . \tag{2}$$

Here, a represents the Fermionic (quasi-particle) component and b is the Bosonic (lattice) term of the heat transport.

By measuring thermal conductivity in a temperature range which is three orders of magnitude lower than the critical temperature and using equation 2 to extract a finite a, solid evidence was provided for the presence of itinerant fermions deep inside the superconducting state[4]. These itinerant Fermions are the nodal quasi-particles of the d-wave gap and their contribution to heat transport extracted using equation 2 is designated here by κ_{00}. This is the electronic T-linear thermal conductivity in the zero-temperature limit.

In 1997, an experiment by Taillefer *et al.*[4] revealed

an intriguing feature of transport by nodal quasi-particles. This feature was first theoretically worked out for microwave conductivity [5] and has been since dubbed universal conductivity. The term *universal* refers to the insensitivity of the magnitude of κ_{00} towards disorder. Indeed, disorder leads to a reduction of the mean-free-path. Thus, it should induce a reduction of conductivity. But, by breaking Cooper pairs, disorder also induces an increase in the density-of-states and provides heat carriers. According to the theory [6-8], for a d-wave gap, these two opposing tendencies compensate each other in such a way that leaves the overall thermal conductivity unaffected. This is why κ_{00} is not to be affected by the introduction of disorder.

To be more specific, according to the theory, κ_{00} is related to the fine details of the electronic energy spectrum. The dispersion in the vicinity of a node can be expressed as:

$$E = \hbar\sqrt{\upsilon_F^2 k_1^2 + \upsilon_2 k_2^2} . \tag{3}$$

Here, $\mathbf{k}_1$ and $\mathbf{k}_2$ are unitary vectors in the reciprocal space. $\mathbf{k}_1$ is normal to the Fermi surface and $\mathbf{k}_2$ is perpendicular to it (See figure 1). υ_F is the more familiar Fermi velocity. υ_2, which is sometimes designated as v_Δ, is called gap velocity. It is proportional to the slope of the d-wave gap, at the node:

$$\upsilon_2 = \frac{1}{\hbar k_F}\frac{d\Delta}{d\phi}|_{node} . \tag{4}$$

The angle ϕ represents the in-plane angle in the reciprocal space. Now, the magnitude of κ_{00} is linked to the ratio of these two velocities in the following way:

$$\kappa_{00} = \frac{k_{B^2}}{3\hbar}\frac{n}{d}(\frac{\upsilon_F}{\upsilon_2} + \frac{\upsilon_2}{\upsilon_F}) . \tag{5}$$

In cuprates, $v_F \gg v_2$ and the second term in the brackets of the right hand side can be safely omitted.

5. Thermal conductivity of cuprates in the zero temperature limit

During the last few years, a finite κ_{00} has been detected in five families of hole-dope cuprates. These are $YBa_2Cu_3O_{7-\delta}$ (Y-123) [4], $Bi_2Sr_2CaCu_2O_{8-\delta}$ (Bi-2212) [10-12], $La_{2-x}Sr_xCuO_4$ (La-214) [13,14], $Tl_2Ba_2CuO_{6-\delta}$ (Tl2201) [15] and $Bi_{2+x}Sr_{2-x}CuO_{6+\delta}$ (Bi-2201) [16,17]. Figure 2, represents the data for three compounds obtained in my group.

The magnitude of κ_{00} obtained in these compounds at optimal doping level is summarized in Table 1. If one assumes that the d-wave gap has the standard angular dependence ($\Delta = \Delta_0 \cos(2\varphi)$), on can extract the magnitude of the maximum gap, Δ_0, which is comparable to what has been directly measured by Angular Resolved Photoemission studies. This indicates that the theory is on the right track.

In two cases, the insensitivity of κ_{00} to the level of disorder was experimentally verified. In the case of Y-123, even after replacing 3 percent of Copper atoms by Zinc, leading to a tenfold decrease in the quasi-particle

Table 1. The magnitude of κ_{00} in three different cuprates. The ratios of two velocities is extracted from κ_{00}. The latter yields the maximum value of the superconducting gap (See text).

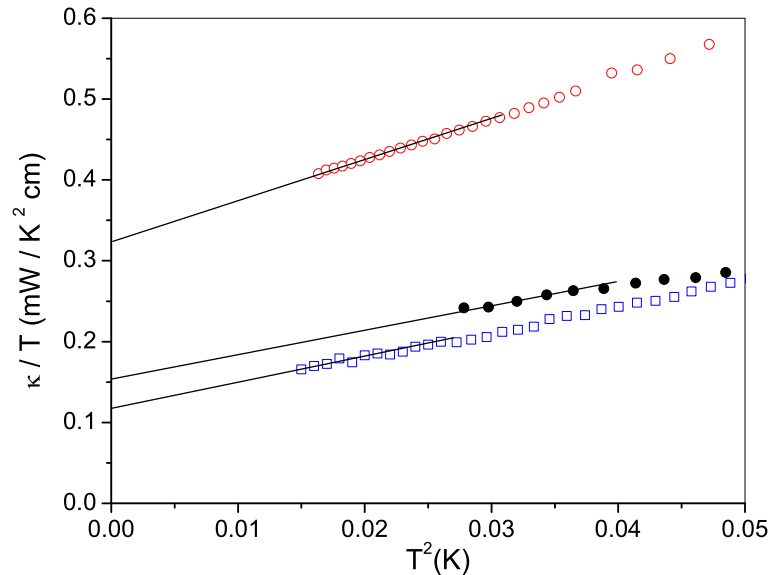| compound	T_c (K)	κ_{00}(mWK^{-1}cm^{-1})	v_F/v_2	Δ_0(meV)
Y-123	91	0.12	12	50
Bi-2212	90	0.19	19	30
Bi-2201	10	0.33	18	8.6

Figure 2. (color online) Thermal conductivity of three optimally-doped cuprates at low temperatures. κ/T is plotted as a function of T^2, in order to separate lattice and electronic contributions. Open squares represent Y-123 , solid circles Bi-2212 and open circles Bi-2201. Lines are extrapolations to zero-temperature and allow to extract κ_{00}.

mean-free-path, no change in the magnitude of κ_{00} was resolved[4]. A similar result was obtained in the case of Bi-2212[12], where disorder was induced using irradiation by high-energy electrons. More recently, Ando and collaborators[19] have reported a finite deviation from universal conductivity in underdoped cuprates which is particularly sizeable in LSCO.

On the other hand, κ_{00}, has been also studied was as a function of doping dependence[13,18]. It was found that it decreases steadily as the Mott insulator is approached. If one assumes that κ_{00} continues to inversely scale with the superconducting gap in the underdoped regime, this result (as argued by Sutherland et al.[18]) points to a superconducting origin for the pseudogap.

The field dependence of thermal conductivity has proved to be interesting too. In the optimally-doped or overdoped cases, κ_{00} increases as a function of magnetic field[20,21], providing an experimental confirmation of Volovik excitations [22] associated with a d-wave superconductor. In underdoped LSCO, however, κ_{00} *decreases* with magnetic field displaying the thermal equivalent of the metal-insulator transition observed by resistivity measurements [14,23].

6. Universal thermal conductivity in other unconventional superconductors

During the past decade, low temperature thermal conductivity of other superconductors was also explored. A finite κ_{00} was observed in the organic superconductor κ-(ET)$_2$Cu(NCS)$_2$ with a magnitude comparable to what is theoretically expected [24]. The d-wave superconductivity in this system remains most plausible. However, it is not the object of a consensus among researchers in the field [25].

Very recently, a finite κ_{00} was reported in CePt$_3$Si [26], which is a heavy-fermion superconductor with a crystal structure with no inversion symmetry. The magnitude of the observed term is in very good agreement with the theoretical expectation, providing evidence in favor of the presence of line nodes in this superconductor.

The most convincing case for universal conductivity has been made for the unconventional superconductor Sr$_2$RuO$_4$[27]. Suzuki et al.[28] reported that the magnitude of κ_{00} is almost insensitive to disorder. Moreover, the slight increase with the change in the scattering rate was found to be in excellent agreement with the theory. This result imposes strong constraints on

the possible symmetries of the superconducting order parameter in this unconventional superconductor. While, it is widely believed to be a triplet superconductor, the

precise identity of the order parameter has not been settled down.

References

1. For a review, see: J Orenstein and A J Millis, Science. (2000) 468.
2. P A Lee, N Nagaosa and X-G Wen, *Rev. Mod. Phys.* **78** (2006) 17.
3. C C Tsuei and J R Kirtley, *Rev. Mod. Phys.* **72** (2000) 969.
4. L Taillefer L, B Lussier R Gagnon, K Behnia and H Aubin, *Phys. Rev. Lett.* **79** (1997) 483.
5. P A Lee, *Phys. Rev. Lett.* **71** (1993) 1887.
6. M J Graf, S–K. Yip, J A Sauls and D Rainer, *Phys. Rev.* B **53** (1996) 15147.
7. A C Durst and P A Lee, *Phys. Rev.* B **62** (2000) 1270.
8. M. R. Norman and P. J. Hirschfeld, Phys. Rev. B **53** , 5706 (1996)
9. Y Sun and K Maki, Europhys. Lett. 32, 355 (1995)
10. K Behnia, S Belin, H Aubin, F Rullier-Albenque, S Ooi, T Tamegai, A Deluzet and P Batail, J Low *Temp. Phys.* **117** (2000) 1089.
11. M Chiao, R W Hill, C Lupien, L Taillefer, P Lambert, R Gagnon and P Fournier, *Phys. Rev.* B **62** (2000) 3554.
12. S Nakamae, K Behnia, L Balicas, F Rullier-Albenque, H Berger and T Tamegai, *Phys. Rev.* B **63** (2001) 184509.
13. J Takeya, Y Ando, S Komiya and X F Sun, *Phys. Rev. Lett.* **88**, (2002) 077001.
14. D G Hawthorn, R W Hill, C Proust, F Ronning, M Sutherland, E Boaknin, C Lupien, M A Tanatar, J Paglione, S Wakimoto, H Zhang, L Taillefer, T Kimura, M Nohara, H Takagi and N E Hussey, *Phys. Rev. Lett.* **90** (2003) 197004.
15. C Proust, E Boaknin, R W Hill, L Taillefer and A P Mackenzie, *Phys. Rev. Lett.* **89** (2002) 147003.
16. R Bel, K Behnia, C Proust, P van der Linden, D Maude and S I Vedeneev, *Phys. Rev. Lett.* **92** (2004) 177003.
17. C Proust, K Behnia, R Bel, D Maude and S I Vedeneev, *Phys. Rev.* B **72** (2005) 214511.
18. M Sutherland, D G Hawthorn, R W Hill, F Ronning, S Wakimoto, H Zhang, C Proust, E Boaknin, C Lupien, L Taillefer, R Liang, D A Bonn, W N Hardy, R Gagnon, N E Hussey, T Kimura, M Nohara and H Takagi,, *Phys. Rev.* B **67** (2003) 174520.
19. X F Sun, S Ono, Y Abe, S Komiya, K Segawa and Y Ando, *Phys. Rev. Lett.* **96** (2006) 017008.
20. H Aubin, K Behnia, S Ooi and T Tamegai, *Phys. Rev. Lett.* **82** (1999) 624.
21. M Chiao, R W Hill, C Lupien, B Popic', R Gagnon, and L Taillefer, *Phys. Rev. Lett.* **82** (1999) 2943.
22. G E Volovik, *JETP Letters* (1993) 469.
23. X F Sun, S Komiya, J Takeya and Y Ando, *Phys. Rev. Lett.* **90** (2003) 117004.
24. S Belin, K Behnia and A Deluzet, *Phys. Rev. Lett.* **81** (1998) 4728.
25. For a review, see : M. Lang and J Mueller, cond-mat/0302157 (2003)
26. K Izawa, Y Kasahara, Y Matsuda, K Behnia, T Yasuda, R Settai and Y Onuki, *Phys. Rev. Lett.* **94** (2005) 197002.
27. For a review, see : A P Mackenzie and Y Maeno, *Rev. Mod. Phys.* **75** (2003) 657.
28. M Suzuki, M A Tanatar, N Kikugawa, Z Q Mao, Y Maeno and T Ishiguro, *Phys. Rev. Lett.* **88** (2002) 227004.