Solar rotation gravitational moments

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Abstract

Gravitational multipole moments of the Sun are still poorly known. Theoretically, the difficulty is mainly due to the differential rotation for which the velocity rate varies both on the surface and with the depth. From an observational point of view, the multipole moments cannot be directly measured. However, recent progresses have been made proving the existence of a strong radial differential rotation in a thin layer near the solar surface (the leptocline). Applying the theory of rotating stars, we will first compute values of J2 and J4 taking into account the radial gradient of rotation, then we will compare these values with the existing ones, giving a more complete review. We will explain some astrophysical outcomes, mainly on the relativistic Post Newtonian parameters. Finally we will conclude by indicating how space experiments (balloon SDS flights, Golf NG, Beppi-Colombo, Gaia...) will be essential to unambiguously determine these parameters.

1. Introduction

There are several motivations for studying the solar gravitational multipole moments. One of the first is to know their accurate values. Using a book on astrophysical quantities, such as Allen [1], one can find the Earth multipole gravitational moments up to the 20th degree in a spherical harmonic expansion. Today, computations have even been made up to the 360th degree [2]. These parameters are also known for some solid planets, such as Mars, up to the 75th degree [3]. In Allen's book considered as a reference – nothing is mentioned for the Sun. The reason is partly due to the different physical structure of the two bodies. For rigid bodies such as the Earth, a uniform velocity rate ! causes a flatness f, and an unambiguous relation links this flatness with the quadrupole moment, J2, at least to the first order. Since the velocity rate is uniform whatever the latitude, it is possible to deduce the density variations inside the body (see for example [4] p.89). Developing the gravitational field into spherical harmonics, the theory of uniform rotating bodies allows to determine deviations from the best ellipsoid (called geoid in the Earth's case1). The artificial satellite era has provided a great amount of data with considerable accuracy and resolution showing gravity anomalies which call for an interpretation in terms of density anomalies in latitude and depth. The result is a distorted Earth shape, in terms of deviations from the geoid. The situation is comparable for the Sun, and maybe also for gaseous planets[1], but inverting parameters: the density can be considered as a monotonously decreasing function from the core to the surface (except within the thin tachocline and leptocline layers for the Sun), whereas the latitudinal rotation velocity rate is no more constant. Rozelot et al. showed for the first time [5,6], this yields departures from a perfect ellipsoid. By analogy with the usually adopted terminology for the Earth, we named the reference gravity equipotential of the Sun the *helioid*, and the departures were called *asphericities* by Armstrong and Kuhn [7]. We usually note asphericities by c_n; these "shape" parameters are nothing else but the successive Denoting by **r** the radius of the fractional shell building progressively the whole Sun and by θ the colatitude, we see that in such a theoretical approach, the three quantities, $\omega(r,\theta)$ (velocity rate), $\rho(r, \theta)$, (density) and c_n (**r**) (shape), are linked together. As helioseismic measurements currently lead to a very precise determination of the different velocity rates on the surface and below, including the sub-surface radial gradient, and as the density function is known with a sufficient accuracy from the core to the surface, it is

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1. Such as Saturn for wich the differential equator to pole rotation is some 14 minutes.

possible to determine the asphericity coefficients, and to compare them to the most accurate observations described hereafter.

2. Relevance of knowing precise solar gravitational moments

For a sake of clarity, let us remind that gravitational moments are the coefficients in the spherical harmonic development of the total gravitational potential (for a complete description, see [6]). The first moments are called

• for n = 1, J2 the solar quadrupole moment
• for n = 2, J4 the solar octopole moment
• for n = 3, J6 the solar dodecapole moment, and so on.

Where **n** is called degree. These parameters are dimensionless quantities providing information on how the mass and velocity distributions act inside the Sun to finally render the outer visible shape non spherical. Higher orders, J8 or J10 for example, are still speculative in the Sun case and we do not deal with them here. The amplitude, peak to peak, of the asphericities does not exceed some 20 milliarcseconds (abbreviated as "mas"), as shown by observations ([6,8] see also section 6). At least three major astrophysical conclusions can be drawn:

1. Gravitational moments are fundamental quantities, just like temperature, mass or luminosity. Directly related to the inertial moments, they should appear in books devoted to solar quantities. However, we have to unambiguously determine their values. Presently, an upper limit, $-3.0 \ 10{-6}$ [9], can be assigned to J2, as any larger observed values would be no more compatible with indirect effects, such as lunar physical librations. The precise estimate is very sensitive to the velocity rotation rate, the density gradient, and the adopted method to solve the question. J2 is certainly bounded by $\approx -2.2 \ 10{-7}$ and $\approx -6.5 \ 10{-7}$ taking into account respectively helioseismic estimates or solar surface tracers (such as spots). Values of higher multipole moments are still not known accurately.

2. One effect of a dynamic flattening of the Sun, due to its oblique rotation with respect to the ecliptic plane (about 7◦), is a secular variation in the orbital elements of the planets which is not negligible, and is often approximated in the computation of ephemerides of the celestial bodies. Modern planetary ephemerides now include a non null value of $\mathbf{J_2}$. However, they are presently not able to infer simultaneously all the usual parameters (masses, radii,..., Post-Newtonian –PN– parameters) and $\mathbf{J_2}$ from fits to observational data, due to strong correlations. The order of magnitude adopted for $\mathbf{J_2}$ corresponds to $10{-7}$, but, for example, the estimation of the PN parameters is rather tolerant to the assumed precise value of $\mathbf{J_2}$.

3. In an attempt to elaborate alternative theories to General Relativity, Brans and Dicke [10] were certainly the first to draw attention on the necessity to determine accurately the solar quadrupole moment 3. Accurate estimations of both the perihelion advance of planets and **Jn** are essential to constrain the PN relativistic theories of gravitation.

3. Effects of the Radial Gradient of Rotation

Whatever the method used, the determination of the gravitational moments require the knowledge of an analytical law describing the non-uniform rotation of the Sun (on the surface and deeper). As soon as 1996, Patern`o *et al.* proposed a quadratic law for which coefficients were empirically determined. With hindsight, it must be noted that the authors had there a true intuition, as their results are close to those obtained by more refined functions. From the BBSO p-modes observations, Kosovichev [12] suggested a rotation law constructed by using error functions *(erf)*, thus permitting to take into account changes in the velocities rates, at the level of the tachocline and near the surface. Based on this original formulation, Dikpati *et al.* [13] developed an analytical law for the solar rotation as a function of latitude and depth. The innovative fact is the near surface radial gradient found to be constant from the equator up to about 20◦ (but negative, _ -400 nHz/R), then progressively inflecting up to 50◦, and finally reversing to be positive above this latitude. The model is based upon a simple equation involving a total of thirteen straightforward parameters. The equation describing the rotation law is related to the Kosovichev's model by assuming "conservation of the angular momentum in the supergranulation layer". The surface radial gradient is expressed by **β**, a polynomial function of the latitude, implying three coef-ficients ($\boldsymbol{\beta_0}$, $\boldsymbol{\beta_3}$ and $\boldsymbol{\beta_6}$). This model is attractive for our purpose, because one can assign different (but realistic) values to six parameters: two stem from the standard rotation law (standard means that the law is expressed as the sum of two terms in cosine of the colatitude), three others describe the radial surface gradient, and the last one is involved in the equatorial velocity rate (other parameters are of less importance).

4. Results and discussion

To solve completely the problem, one needs a solar density model [12]. Thanks to years of improvements in solar modeling, several models are available. We used the Richard model [14]; a discussion of solar seismic models can be found in Couvidat *et al.* [15]. To be coherent with law of rotation given in Rozelot and Lefebvre [6], we adopted $\mathbf{a_2} = 0.442$, $\mathbf{a_4} = 0.056$ and $\boldsymbol{\omega_{pol}} = 2.399$ (all units in $\boldsymbol{\mu rad/s}$). We found: $\mathbf{J_2} = -2.5 \ 10{-7} \pm 20\%$ and $\mathbf{J_4} = +4.5 \ 10{-7} \pm 25\%$.

The range of the results comes from the different values assigned to the six above mentioned parameters. It is thus demonstrated that the two first solar gravitational moments, and mainly $\mathbf{J_4}$, are very sensitive

to what happens in the sub-surface. We suspect an important role of both the density gradient and the radial gradient of rotation.

4.1. Comparison with other values
4.1.1. As derived from stellar structure equations

Ulrich and Hawkins [28], taking into account a standard rotation law, derived an estimate of J_2 bounded by 1.0 10−7 and 1.5 10−7. They also found that J_4 must be of the order of J_2/40. One year latter, using helioseismic rotation rates, Gough [29] obtained J_2 = 36 10−7, a value altogether compatible with those deduced from astrometry by Campbell and Moffat [16] or Landgraf [17], i.e. (5.5 ± 1.3) 10−6 and (0.6 ± 5.8) 10−6 respectively.

To this must be added the result obtained by Patern`o et al. [11] using a value of the solar oblateness deduced from the SDS experiment, i.e. f = 8.63 10−6, leading to J_2 = 2.22 10−7.

Godier and Rozelot [18] using the Kosovichev rotation law found 1.60 10−7, a possibly underestimated value (if we consider the different results known to date), certainly due to truncatures in the integration process. Roxburgh [19] reconsidered this process (but using other density models), and obtained J_2 = 2.208 10−7 and J_4 = -4.46 10−9. Along the same principles, Mecheri et al. [20] found J_2 = 2.2 10−7 and J_4 = -4.8 10−9

In order to avoid truncatures, Armstrong and Kuhn [7] developed a "Vector Spherical Harmonic" expansion that leads to J_2 = -0.222 10−6 and J_4 = 3.84 10−9. - Finally, Pijpers [21] using GONG and SOI/MIDI data found respectively J_2 = (2.14±0.09) 10−7 and (2.23±0.09) 10−7, leading to a weighted value of (2.18±0.06) 10−7.

4.2. As derived from the theory of Figures

An interesting alternative to the theory of stellar structure is the theory of Figures of rotating bodies. The method is widely explained elsewhere [6]. Values obtained for the gravitational moments with this method are significantly higher than those derived from the method cited in the above paragraph. A factor three is obtained in the case of J2 and a factor hundred in the case of J4. It must be noted, however, that both theoretical values deduced from the theory of Figures are compatible, within a factor two, with values deduced from observations [22]: J2 = 1.84 10−7 and J4 = 9.83 10−7. Presently, we do not have a clear explanation to this. Initially, we believed that the discrepancy could come from using a rotation law which does not derive from a potential [5]. It is why we adopted another rotation law, but the results are not appreciably different. Moreover, this new rotation law prevents a true comparison with the a_i rotation coefficients used by other authors. However, the theory of Figures is incredibly accurate in the case of the Earth and solid planets. Why should it not work for the Sun?

4.3. Partial conclusion

To sum up the discussion, we would say:
1. The theory of rotating stars provides values of solar gravitational moments that are very dependent upon the differential rotation and the density gradient (mainly near the surface).
2. The theory of Figures of rotating bodies provides values of solar gravitational moments closer to those observed.
3. Matching the two theories remains to be made. It is clear that measurements of both J2 and J4 would provide relevant constraints on $\omega(r, \theta)$, which are different from the ones imposed by solar oscillations. Such measurements have to be made from space.

5. Relativity inferences

Pireaux and Rozelot [23], then Rozelot et al. [24] have already pointed out how accurate observations of the perihelion advances of Mercury and minor planets (such as Icarus) together with accurate values of the J_n may help to constrain the Eddington-Robertson parameters _ and in the PN parameterization formalism, which characterizes alternative theories of gravitation. The authors have shown that such theories are not excluded (see for instance Fig. 3 in [24]). Future space missions, or solar probes, are necessary to conclude.

6. Shape Coefficients

To our knowledge, measurements of the shape coefficients have only been made from space by Armstrong and Kuhn [7] and from the ground by [8]).

Thanks to accurate measurements from the SOHO/MDI satellite experiment Armstrong and Kuhn [7] revisited the multipole shape terms of order higher than the oblateness. They found that the two first shape coeffi-cients c_2 and c_4 are: c_2 = (-5.27 ± 0.38) 10−6 and c_4 = (+1.3 ± 0.51) 10−6. One of their main conclusions is that the quadrupole and octopole terms are inconsistent with the present solar rotation data. As far as ground experiments are concerned, the scanning heliometer used at the Pic du Midi observatory has been described for instance in [25] or [8]. By means of this instrument, we observed in 1996 and 1997, which were mainly years of development and improvements of the instrument. We measured mostly equatorial and polar diameters of the Sun and made a very few other measurements at different heliographic latitudes. The reason is the time-consuming way data are acquired, by passing successive cords on both sides of the diameter to make sure to measure a "true" diameter at a given heliographic latitude. In 1998 and 1999, no data were acquired as the observatory was closed for works. Each first week of September since 2000, a routine campaign is made. Exceptional weather conditions were encountered in 2001: moderate North-West wind, seeing at the diffraction limit of the refractor, around 20 to 25-cm 4. As a complete set of
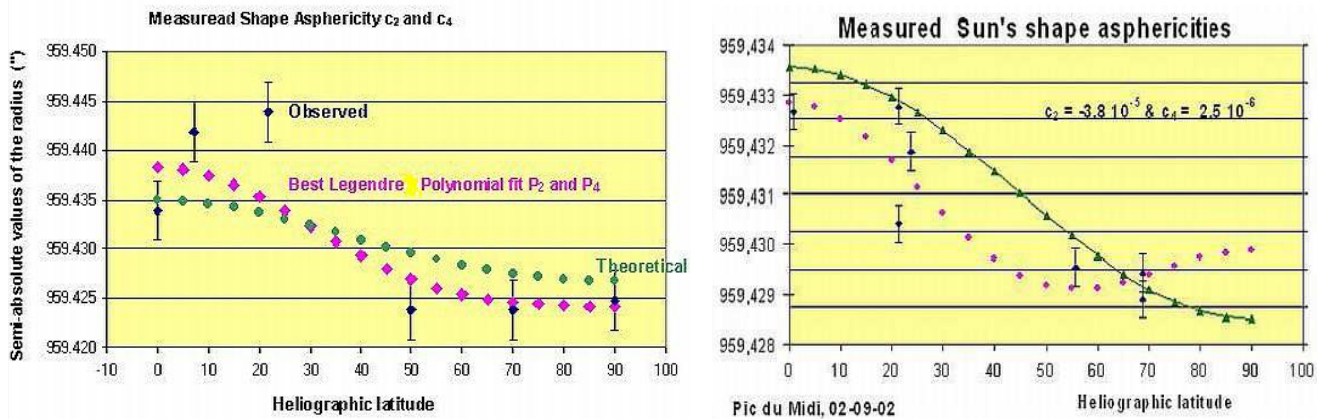
Figure 1. Solar Asphericities deduced from measurements made by means of the scanning heliometer at Pic du Midi Observatory in 2001 (left) and (right). On the left plot, values of c2 and c4 derive directly from an expansion of the diameter in Legendre polynomials, whereas on the right plot, these cn come from a fit to the best ellipsoid passing through the measured points.

heliographic latitudes is necessary to infer shape coefficients, only measurements since year 2000 can be used (however years 1996 and 1997 were used to determine the oblatness). Some results are shown in figre 1. Preliminarily results are:

Year 2000: $c_2 = (-7.6 \pm 0.2) \ 10-6$ and $c_4 = +2.2 \ 10-6$
Year 2001: $c_2 = (-1.1 \pm 0.5) \ 10-5$ and $c_4 = +3.4 \ 10-6$
Year 2002: $c_2 = (-3.8 \pm 0.8) \ 10-5$ and $c_4 = +2.5 \ 10-6$

Differences in the estimates stem from the difficulty of ob- servations, mainly due to seeing conditions. The mean deduced value of c_2, $-7.5 \ 10-6$, is not too far from the theoretical one for a uniform rotation law, $\cong -(2/3)f \cong - 5.9 \ 10-6$ with $f = 8.9 \ 10-6$. The coefficient c_4 remains difficult to match with this theory, which predicts $+(12/35) \ f^2$. The only explanation is that the distorted shape coefficient c_4 is very sensitive to surface phenomena, rotation and a likely steep density gradient. To close this section, let us mention that ground-based solar-astrolabe observations have already lead to similar conclusions. However the obtained shape differs slightly from our results. This may come from the astrolabe measurements themselves, noised by atmospheric turbulence: in the majority of cases, the Fried parameter is only of the order of a few cm. In contrast, a complete analysis of some 30 years of data obtained at Mount Wilson Observatory, by an other method, show very strong similitude with our own measurements [26]. It would be unlikely that results obtained by different techniques leading to comparable outcomes would not draw on a common solar origin.

7. Perspectives

To go further, space experiments are necessary. In the setting of already planned missions such as GAIA or Beppi Colombo, estimations of the solar gravitational moments are envisaged as a by-product. This can be achieved for example in the case of GAIA by measuring the precession of minor planets. Other missions, such as SDO, foresee an accurate measurement of the solar differential rotation both at the surface and in depth. To date, the suitability of the latter measurements on a platform such as Golf-NG is discussed. Nevertheless, all these missions are still far: 2008, 2010 and even 2020. Meanwhile, it is proposed to reactivate SDS (Solar Disk Sextant) flights [27] within an American-European program. One flight each year up to 2007 should provide reliable data over a suitably long period of time. These flights will also allow to extend the database of the previous SDS results and enable us to accurately determine at least the first gravitational moments J2 and J4 (higher moments might be also accessible).

8. Conclusion

This study emphasizes the need to better determine solar gravitational moments. We have also pointed out: 1. large discrepancies in the values obtained for J4. 2. Progress in Jn estimates will depend on a better knowledge of the subsurface adiabatic layer (that we called the leptocline). Space measurements are undeniably needed. Waiting for the advent of scheduled missions, but bearing in mind that the first one is as far as 2008, we urge the scientific community to support balloon flights such as SDS.

References

1. C Allen, *Astrophysical Quantities*, A Cox, ed., Springer (2000).
2. R G Biancale *Geophys. Res. L.*, **27**, 22 (2000) 3611.
3. D-N Yuan W L Sjogren A S Konopliv A B Kucinskas, *J. Geophys. Res. E*, **106** (2002) 23377.
4. E C Bullard The interior of the Earth, in *"The Earth as a planet"*, G P Kuiper ed., Univ. of Chicago

Press, (1954) 57-137.

5. J P Rozelot S Godier and S Lefebvre, *Solar Phys.,* **198** (2001) 223.

6. J P Rozelot and S Lefebvre, in *"The Sun's Surfaceand Subsurface*, Rozelot, ed., LNP 599, Springer, (2003) 4.

7. J Armstrong and J R Kuhn, *ApJ*, **525** (1999) 533.

8. J P Rozelot S Lefebvre and V Desnoux, *Solar Phys.*, **217** (2003) 39.

9. J P Rozelot and E Bois, *19th NSO Sacramento Peak Summer Workshop,* New Mexico (Sept. 1997), published in *"Synoptic Solar Physics"'*, ed. K S Balasubramaniam, et al., *J.A.S.P. Conference Series,* **140** (1998) 75.

10. C Brans and R H Dicke, *Phys. Rev.* **124** (1961) 925.

11. L Patern`o S Sofia and M P Di Mauro, *Astron. & Astrophys.,* **314** (1996) 940.

12. A G Kosovichev, *ApJ*, **469** (1996) 61.

13. M Dikpatj T Corbard, M J Thompson and P A Gilman, *ApJ*, **575** (2002) L41.

14. O Richard, *Ph. D. Thesis, Universit´e Pierre et Marie Curie* (Paris 6), (1999) 149.

15. S Couvidat S Turck-Chieze and A G Kosovichev, *ApJ*, **599** (2003) 1434.

16. L Campbell and J W Moffat,*ApJ* , **275** (1983) L77.

17. W Landgraf, *Solar Phys.,* **14** (1992) 403.

18. S Godier and J P Rozelot, *Astron. & Astrophys.*, **350** (1999) 310.

19. I W Roxburgh, *Astron. & Astrophys.*, **377** (2001) 688.

20. R Mecheri, *et al., Sol. Phys.,* in press (2004).

21. F P Pijpers, *Monthly Notices Royal Astron. Soc.,* **297** (1998) 76.

22. T J Lydon and S Sofia, *Phys. Rev. L.,* **76** (1996) 177.

23. S Pireaux and J P Rozelot, *Astron. & Space Science*, **284** (2003) 1159 (available also on www: astroph/0109032).

24. J P Rozelot, S Pireaux, S Lefebvre and A Ajabshirizadeh, *Proc. Royal. Astr. S.,* submitted (2004), (available also on www: astro-ph 040382).

25. J R¨osch, J P Rozelot, H Deslandes and V Desnoux, *Sol. Phys.,* **165** (1996) 1.

26. S Lefebvre, L Bertello, R K Ulrich, J E Boyden and J P Rozelot (2004).

27. E Maier L Twigg and S Sofia, *ApJ*, **389** (1992) 447.

28. R K Ulrich and G W Hawkins, a: *ApJ*, **246** (1981) 985. *and erratum*, b: *ApJ*, **249** (1981) 831.

29. D O Gough, *Nature*, **298** (1982) 334.