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## Analytical solution of the evolution equations of the parton distribution functions in the small-x region through the Kramers-Moyal expansion of the master equation of Markov processes

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## **Abstract**

Recently, we generated the evolution equations of the parton distribution functions (PDF) usually used in the hadrons phenomenology using the stochastic modeling of the non-equilibrium statistical mechanics in the momentum space. The evolution equations obtained from stochastic modeling are the same as the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations, but are obtained by a more simplistic mathematical procedure based on the non-equilibrium statistical mechanics and the theory of Markov processes. In this paper, we analytically solve the parton evolution equation for the non-singlet quark distribution function in the small-x (the longitudinal momentum fraction) region through the Kramers-Moyal expansion of the master equation. Finally, we compare the cutoff dependent non-singlet quark distribution function obtained from the analytical solution by considering the strong ordering and the angular ordering constraints with the ordinary non-singlet quark distribution function produced by the MMHT2014 group. In general, we show that our results at the small x and moderate  $Q^{\tau}$  (the energy scale) are in good agreement with the results of the MMHT2014 group.

Keywords: parton distribution function (PDF), stochastic modeling, master equation, Markov process, Kramers-Moyal expansion

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