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## Spherical symmetry solutions of Einstein's equations via invariants of symmetry groups

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### Abstract

The Schwarzschild solution was obtained in 1916 as the first solution to Einstein's field equations based on spherical symmetry, which was considered a new idea in the use of symmetry at the time. In this study, it is tried to reformulate the solutions of Einstein's field equations with spherical symmetry in both the presence and absence of the cosmological constant based on the invariants (or first integrals) of the symmetry groups. The method used in this article is a combination of four well-known symmetries: Lie point symmetry,  $\lambda$ -symmetry, Darboux polynomials, and the extended Prolle-Singer method. In this method, to resolve the Schwarzschild problem, a combination of the mentioned symmetries is used in such a way that the independent invariants are obtained in a systematic and algorithmic way. Based on this, a symmetry theory is provided for us. With the help of this theory, one can use symmetries in the best possible way, far from any confusion, and achieve the desired solutions with a specific solution, which is to find independent invariants.

**Keywords:** Lie point symmetry, Darboux polynomials

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For full article, refer to the Persian section