Iranian Journal of Physics Research, Vol. 26, No. 1, 2026
DOI: 10.47176/ijpr.26.1.82170

Heteroclinic attractor in a three-population network of Kuramoto oscillators

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(Received 17 August 2025 ; in final form 06 December 2025)

Abstract
Heteroclinic cycles provide a powerful model for describing transient and flexible sequential state transitions in dynamical systems, with applications ranging from neural networks and genetic circuits to chemical oscillators and lasers. They provide a framework for representing metastable states and analyzing information dynamics. In this study, we consider a three-population Kuramoto-type phase oscillator network in the “near-cluster” regime and, using a reduction onto the Ott–Antonsen manifold, derive a Lotka–Volterra–type competitive system that describes its reduced dynamics. This mapping enables us to obtain explicit necessary and sufficient conditions for the existence of a heteroclinic cycle with a prescribed orientation between the boundary equilibria. These conditions depend directly on the effective coupling coefficients, locked phase differences, and the phase-lag parameter. To assess their validity, the entire space of locked phase differences is sampled and the regions supporting a heteroclinic cycle are identified. Phase-space simulations further confirm the emergence of heteroclinic cycle dynamics. Moreover, the results show that, within a limited range of parameters, increasing the phase-lag constant reduces the size of the phase-difference regions that support the heteroclinic cycle. Taken together, these findings provide testable criteria for designing controllable networks with flexible transient dynamics and establish a clear link between physical oscillator models and neural function, multi-stage rhythm generation in biological systems, and brain-inspired applications in artificial intelligence.

Keywords: Kuramoto oscillator, Lotka–Volterra dynamics, Heteroclinic cycle, Winnerless competition, Transient dynamics

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